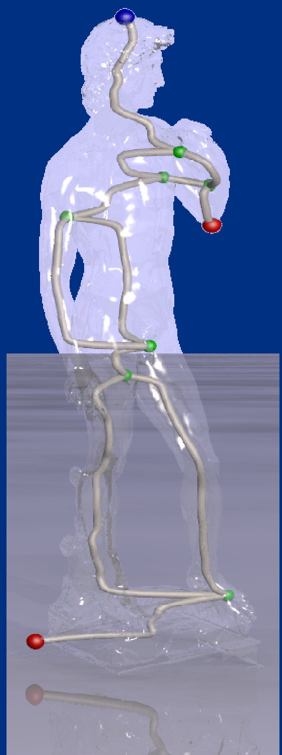
SIGGRAPH2007

Robust On-line Computation of Reeb Graphs: Simplicity and Speed

SIGGRAPH2007



Valerio Pascucci

Giorgio Scorzelli

Peer-Timo Bremer

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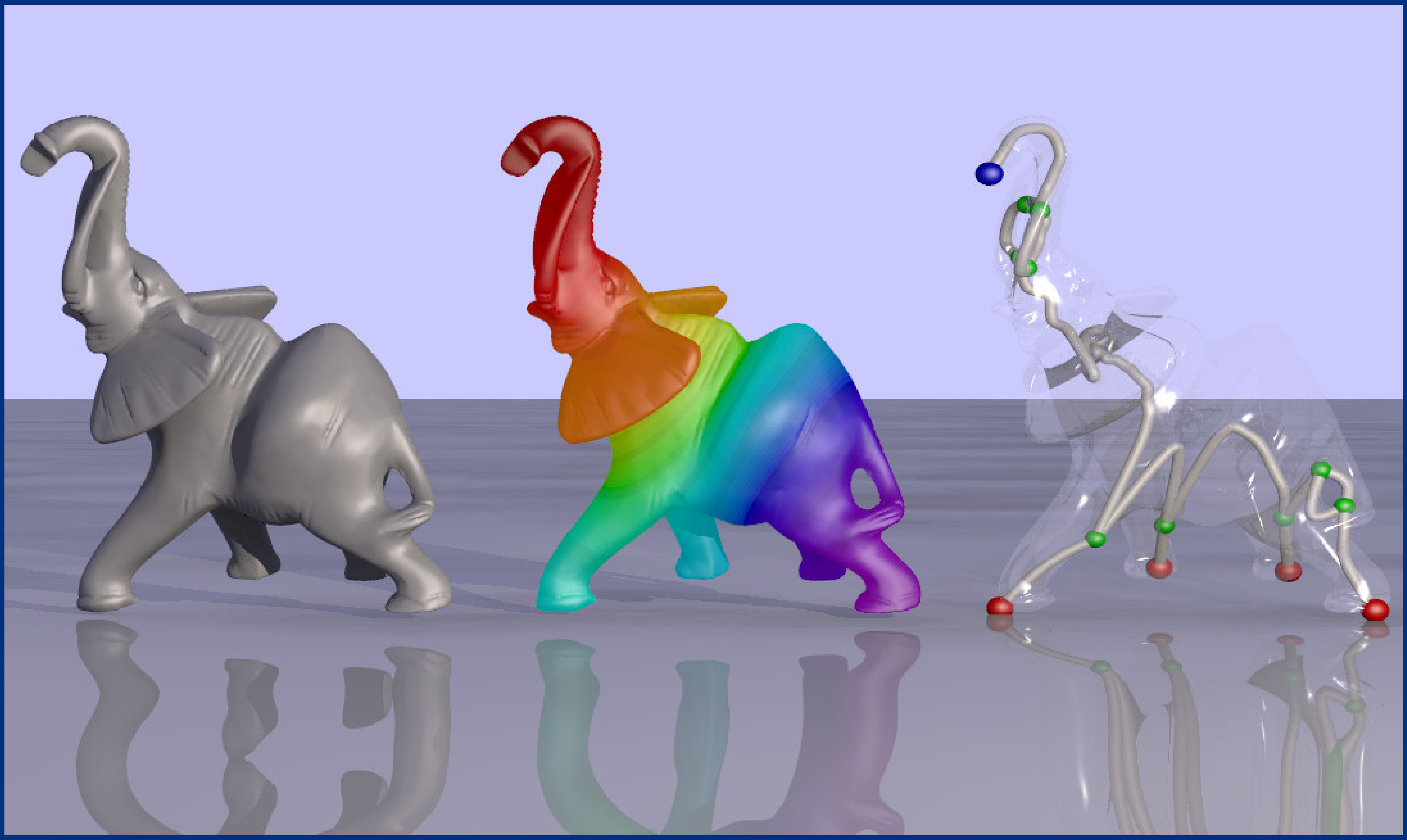
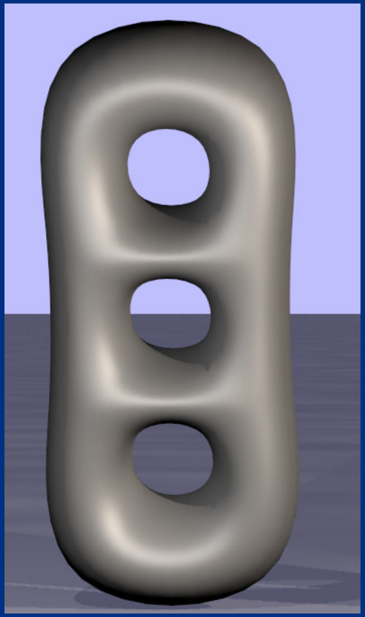
CASC - LLNL

The Reeb Graph Is the Topological Skeleton of a Geometric Model



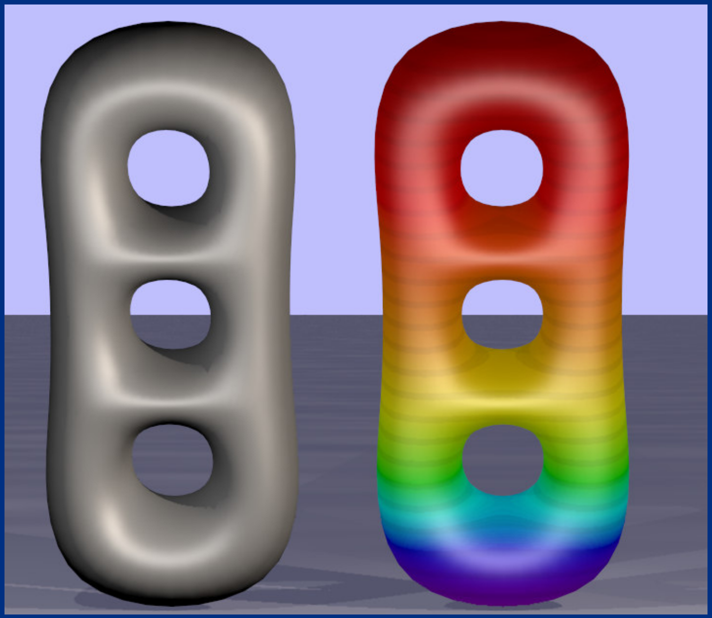
The Reeb Graph Is the Contraction of Isocontour Components to Points

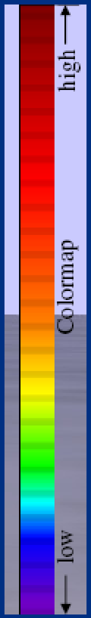
- Given a mesh.



The Reeb Graph Is the Contraction of Isocontour Components to Points

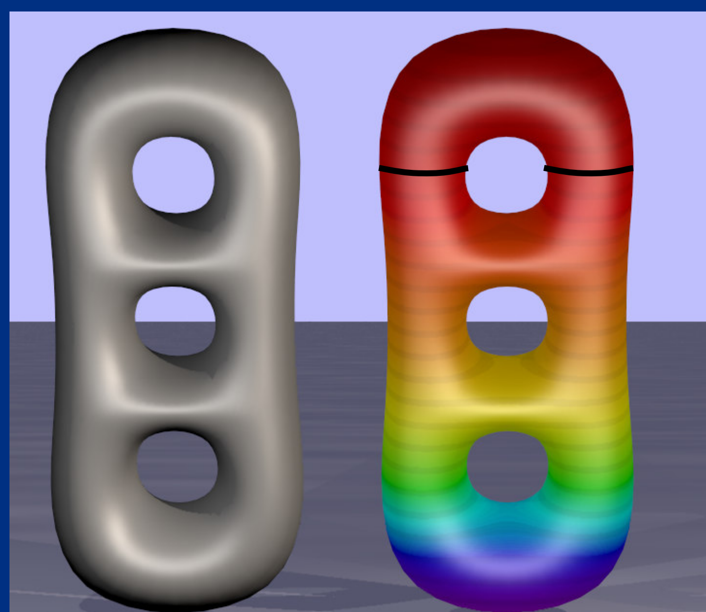
- Given a mesh and a function defined on it.

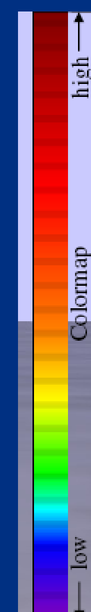




The Reeb Graph Is the Contraction of Isocontour Components to Points

- Given a mesh and a function defined on it.
- Consider an isocontour.





The Reeb Graph Is the Contraction of Isocontour Components to Points

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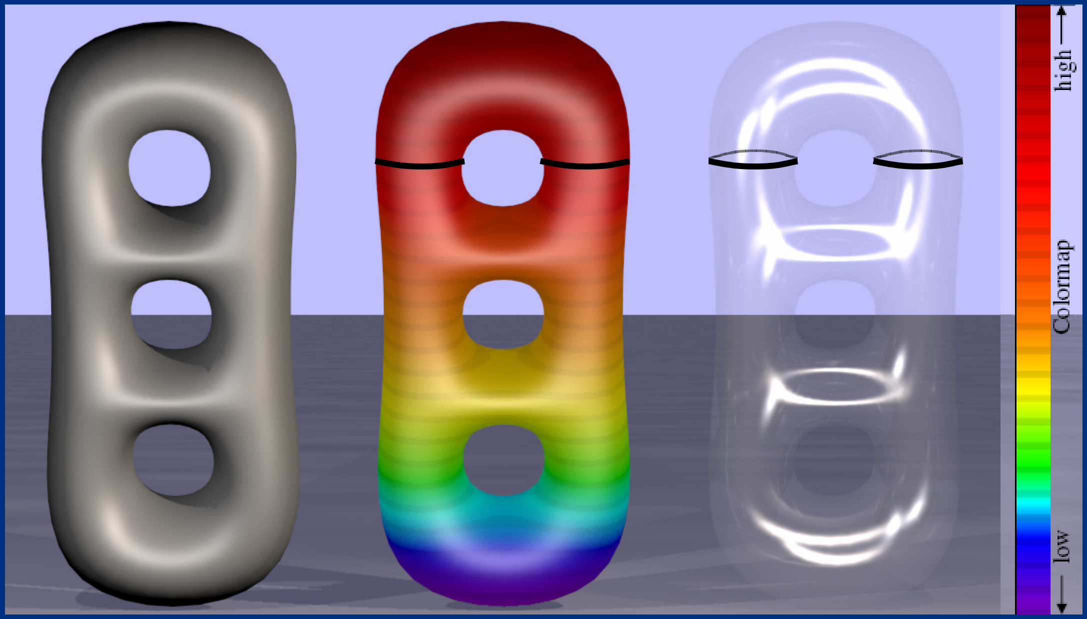
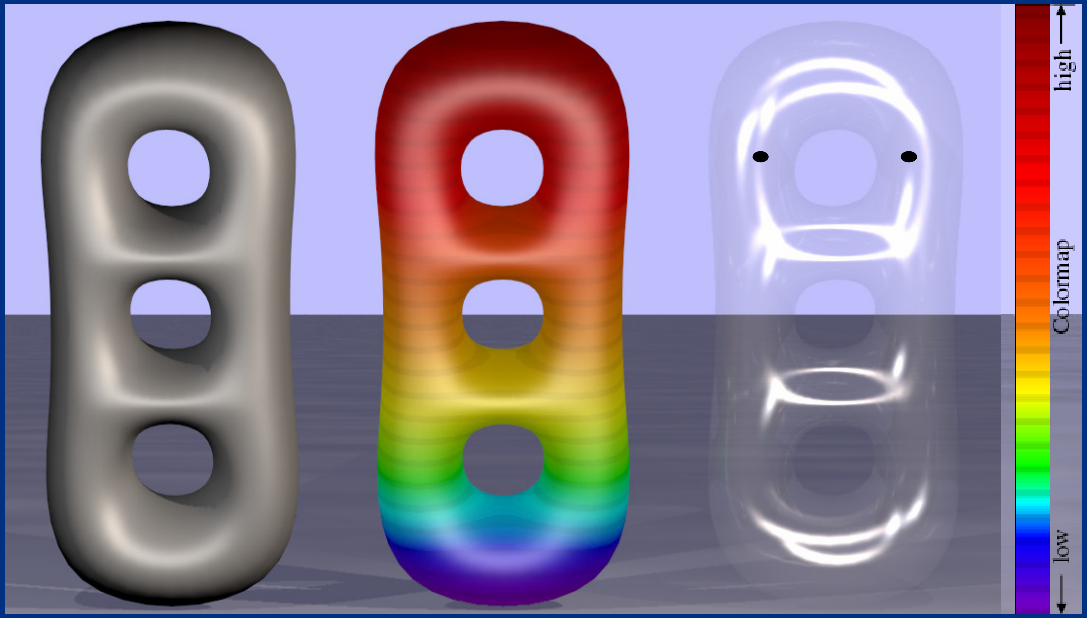


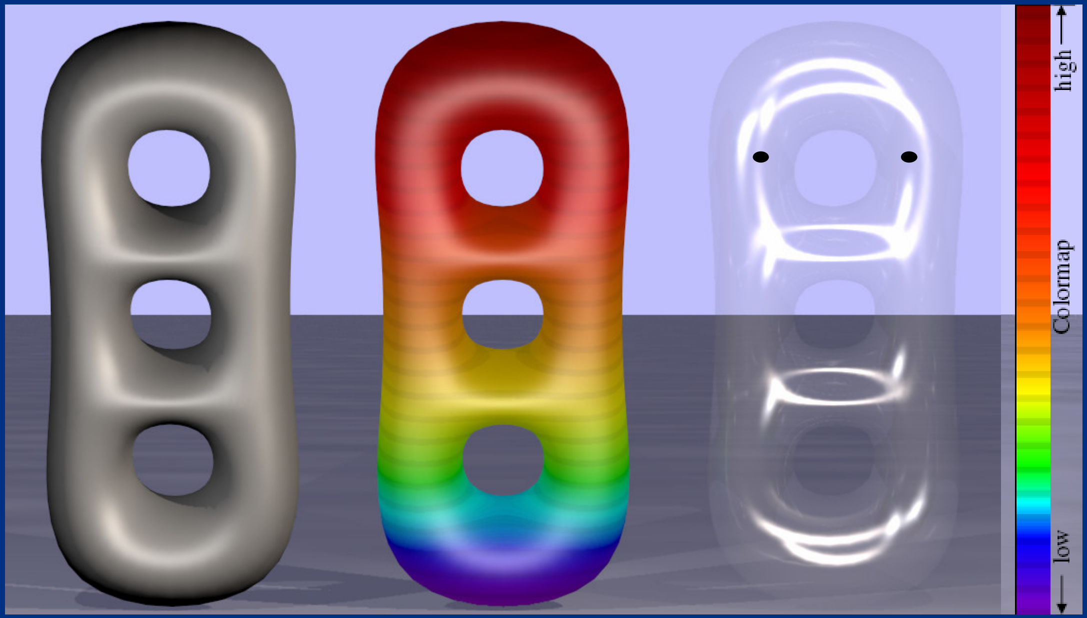
The Reeb Graph Is the Contraction of Isocontour Components to Points

- Given a mesh and a function defined on it.
- Consider an isocontour and contract each component.



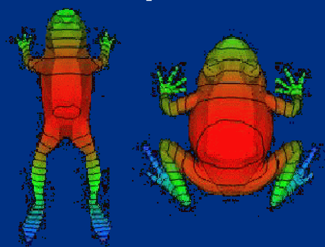
The Reeb Graph Is the Contraction of Isocontour Components to Points

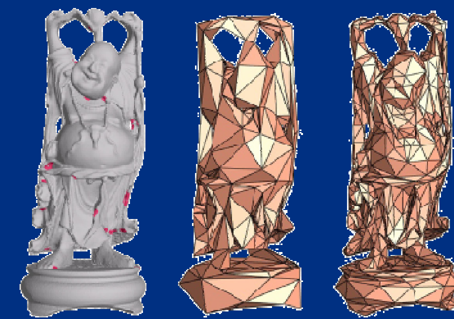
- Given a mesh and a function defined on it.
- Consider an isocontour and contract each component.
- Repeat for all contours while maintaining adjacency.



The Reeb Graph Is a Fundamental Descriptor of the Topology of Shapes

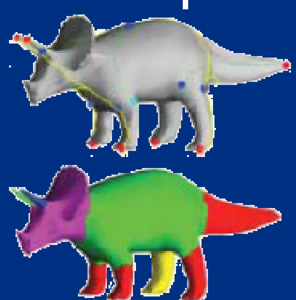
Shape matching [Hilaga et al. SIGGRAPH‘01]

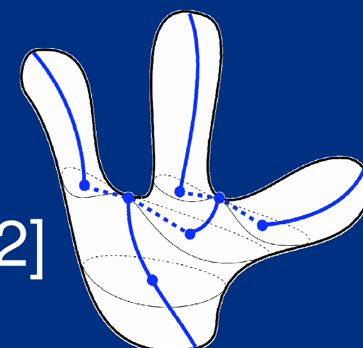




Mesh repair [Wood et al. ToG’04]

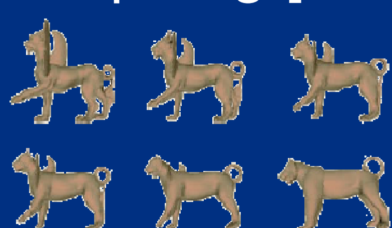
Mesh parameterization [Zhang et al.ToG’05]

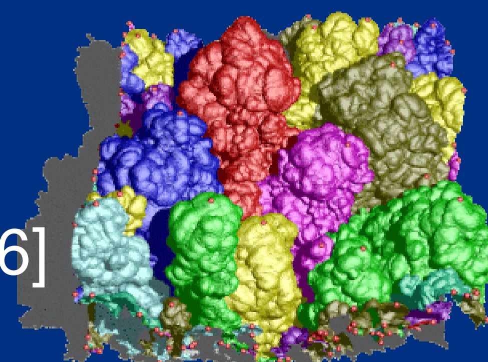




Shape skeletons [Lazarus et al. SM’02]

Morphing [Lee et al. CA&VW ’06]

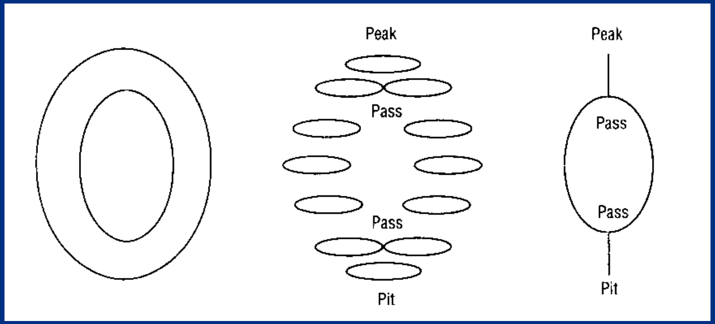


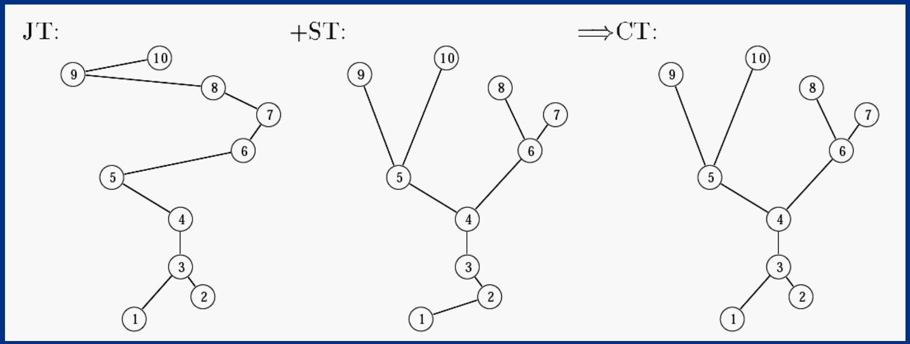


Data analysis [Laney et al. Vis‘06]

Previous Methods Trade Generality for Better Worst Case Complexity

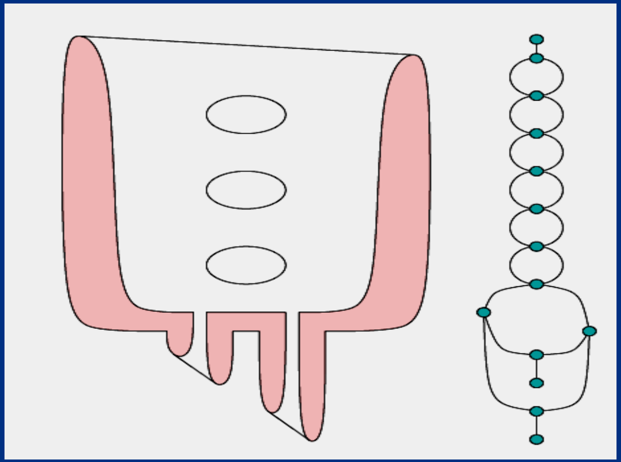
- Construction from slices
  [Shinagawa et al. CG&A'91]





- Contour Trees
  [Carr et al. SODA '00]

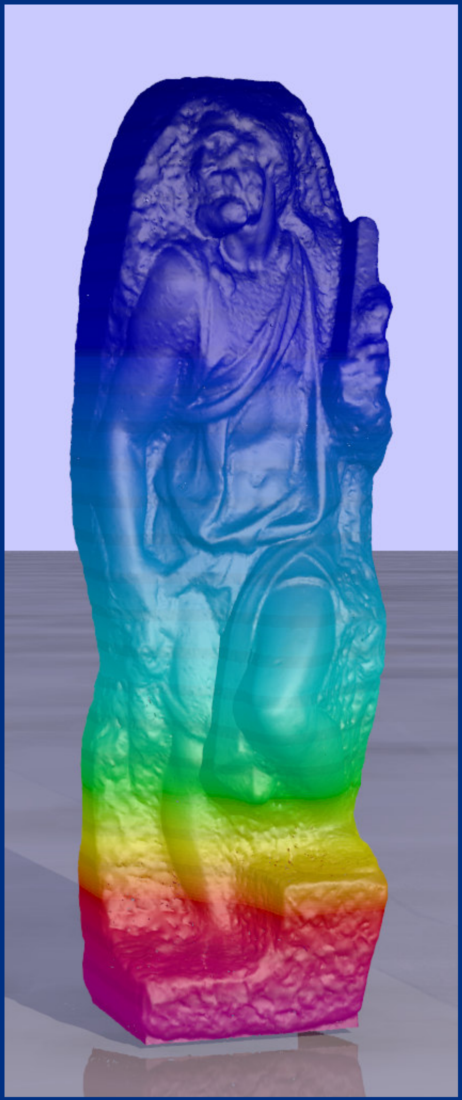
- Loops in Reeb graphs
  [Cole-McLaughlin SoCG'03]



We Propose an Approach Aiming at Generality and Practical Performance

- Input model can be non-manifold.
- Input model of any dimension.

- No need to reorder the input triangles.
- Run in out of core mode.
- Multi-resolution representation.



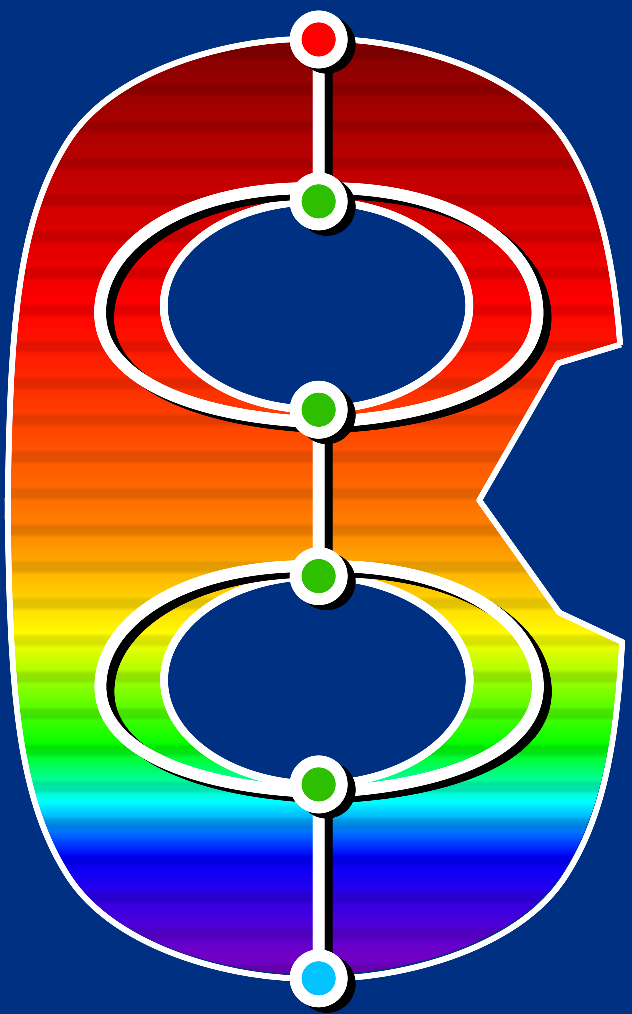
St.Mathew 372Mt, 486s, 8MB.

Update of the Reeb Graph After Insertion of Each New Element

- Consider a 2D model of known Reeb graph (initial condition = empty graph):

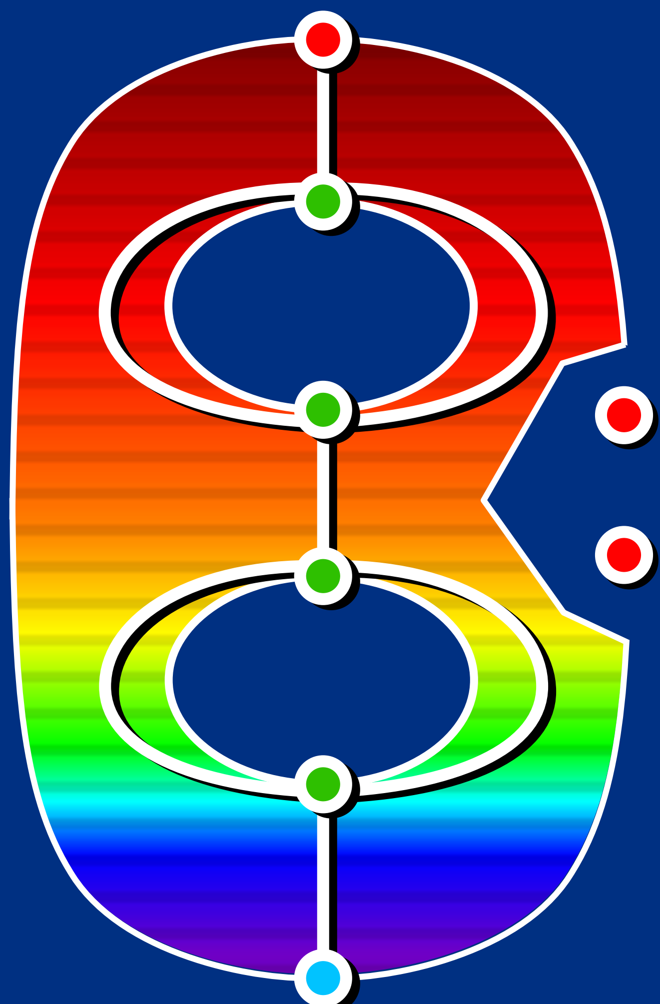
  - Add new vertex.
  - Add new edge.
  - Add new triangle.

On-line Update of the Reeb Graph After Insertion of Each New Element

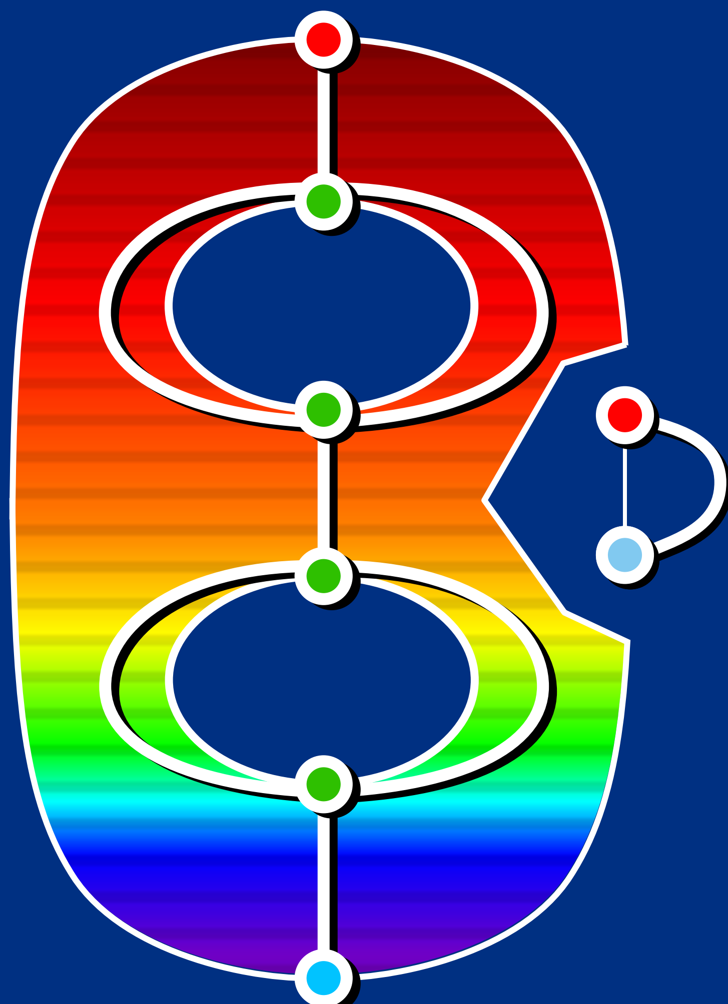


- maximum
- saddle
- minimum

On-line Update of the Reeb Graph After Insertion of Each New Vertex



On-line Update of the Reeb Graph After Insertion of Each New Edge



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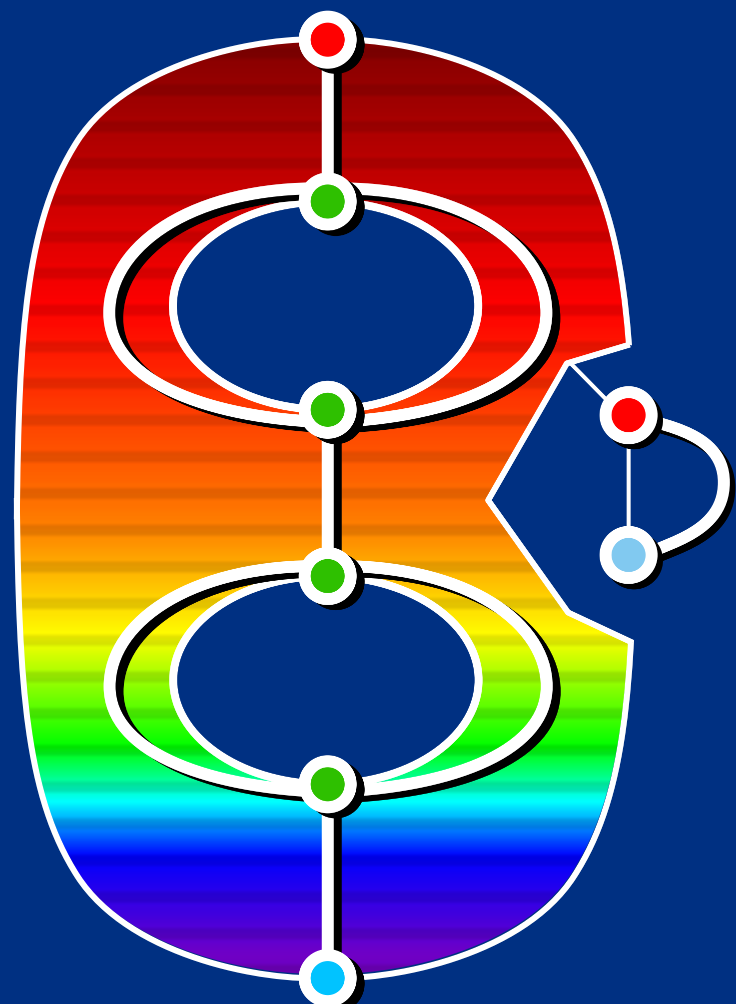

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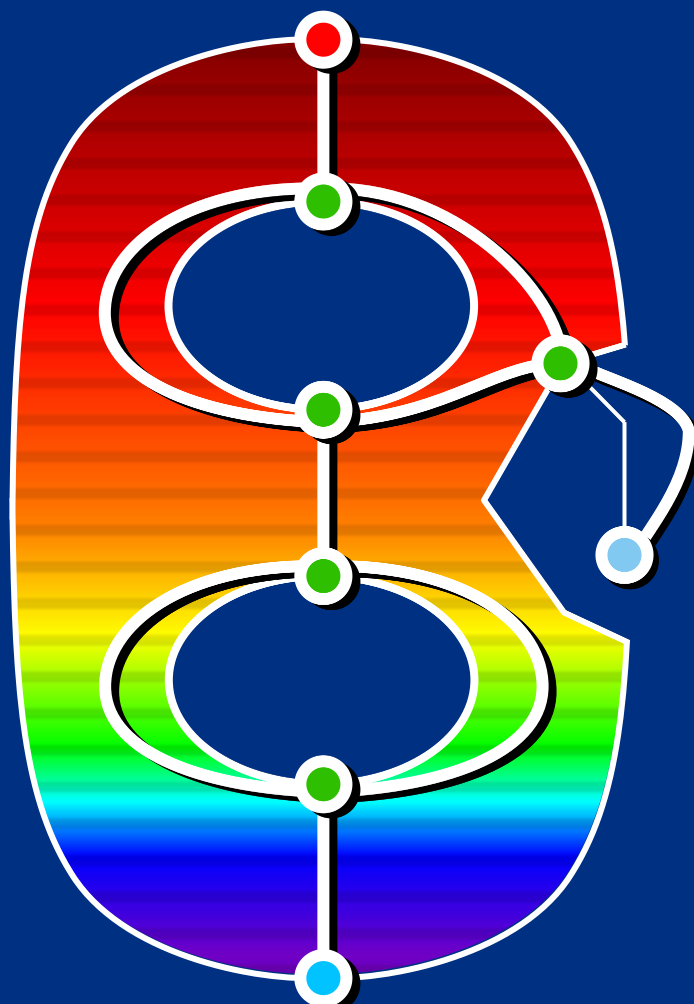

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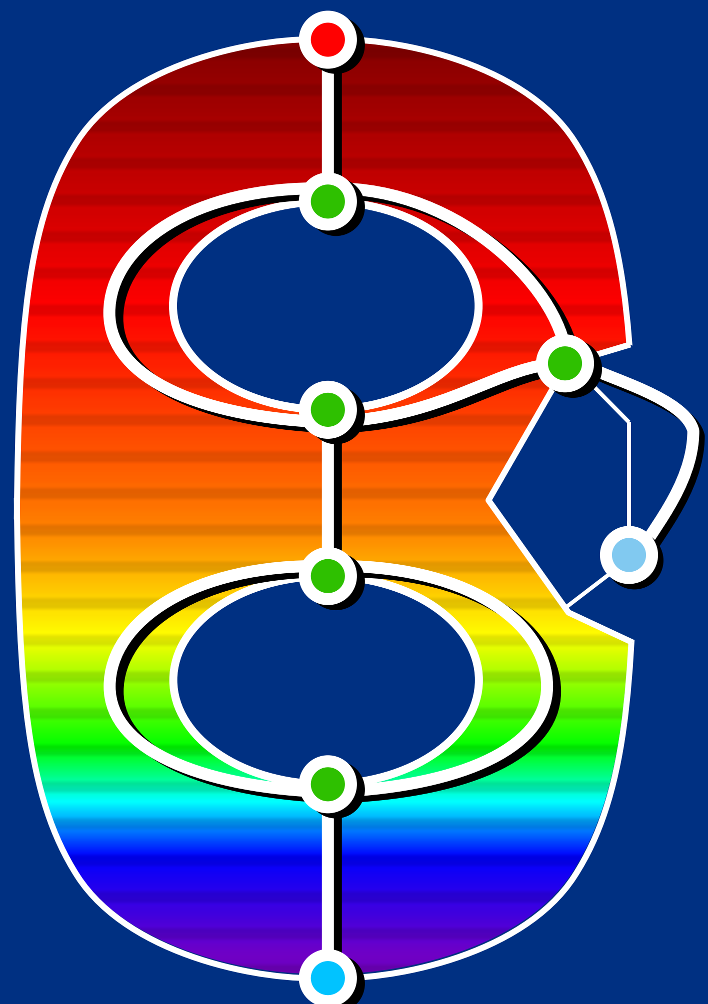

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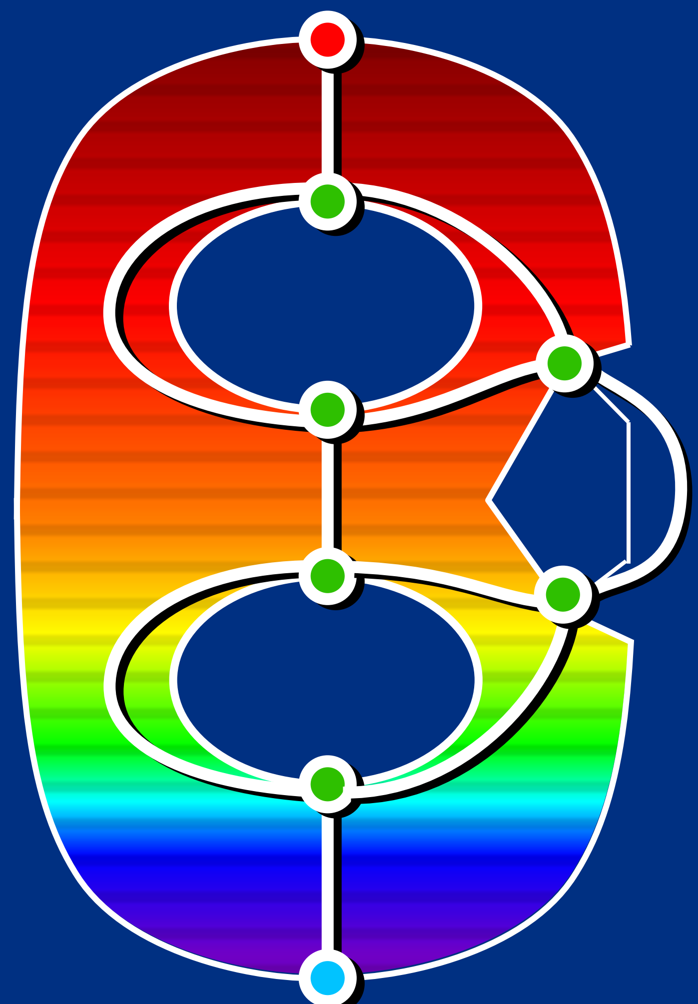
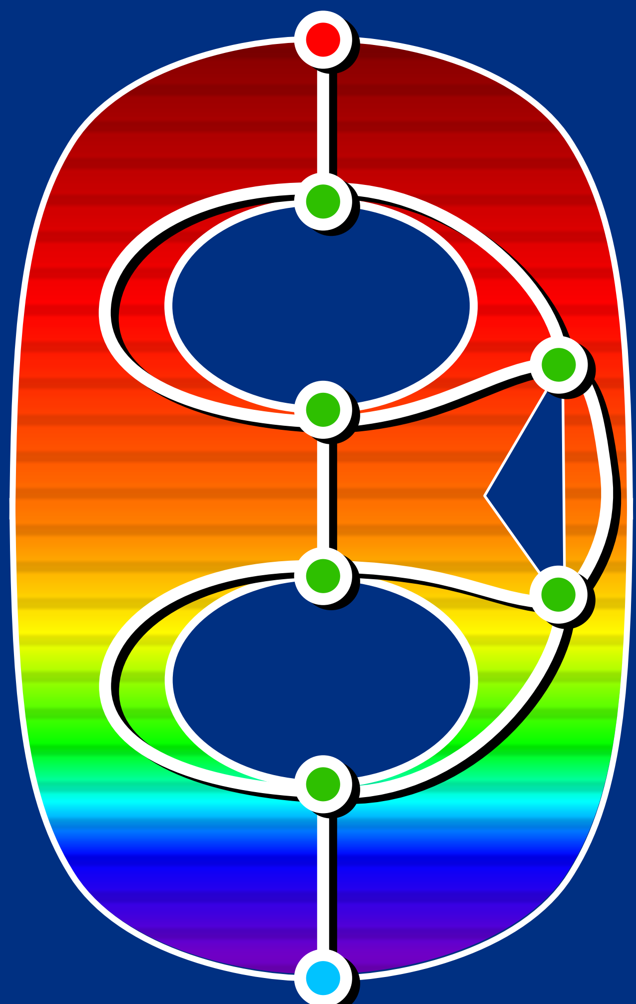


On-line Update of the Reeb Graph After Insertion of Each New Triangle



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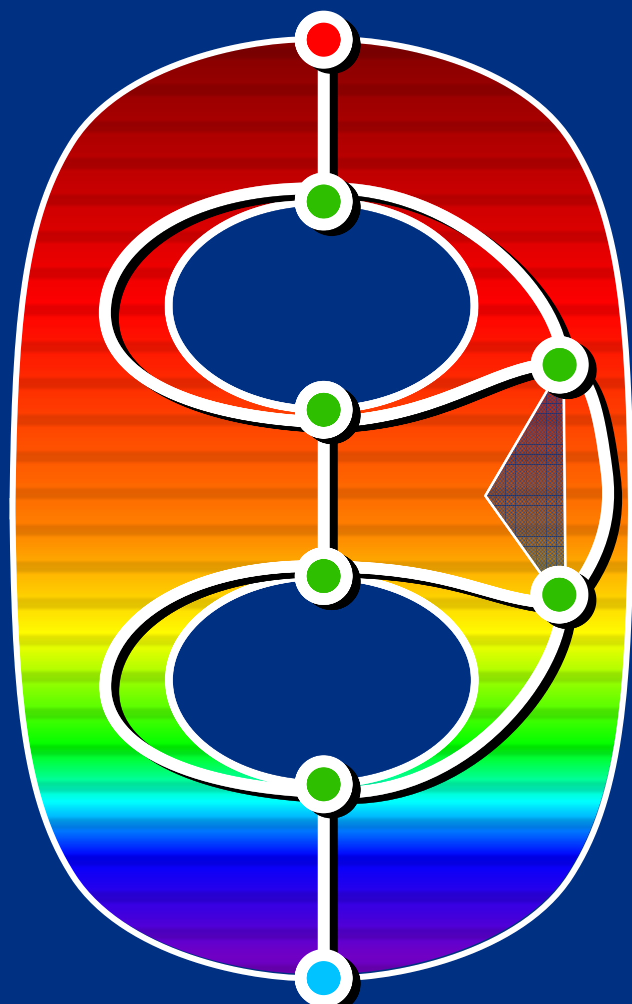
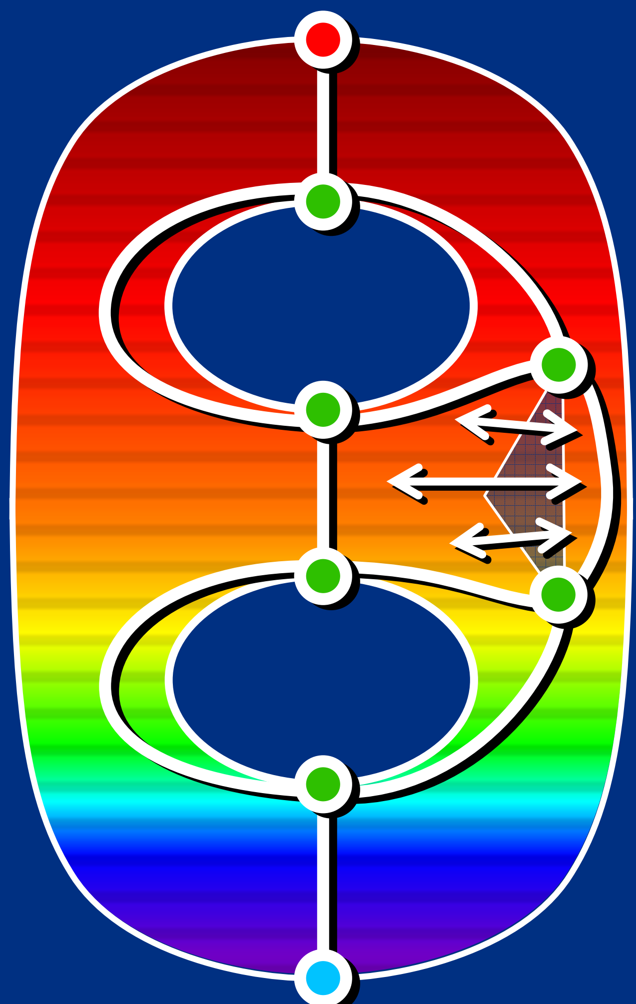


On-line Update of the Reeb Graph After Insertion of Each New Triangle



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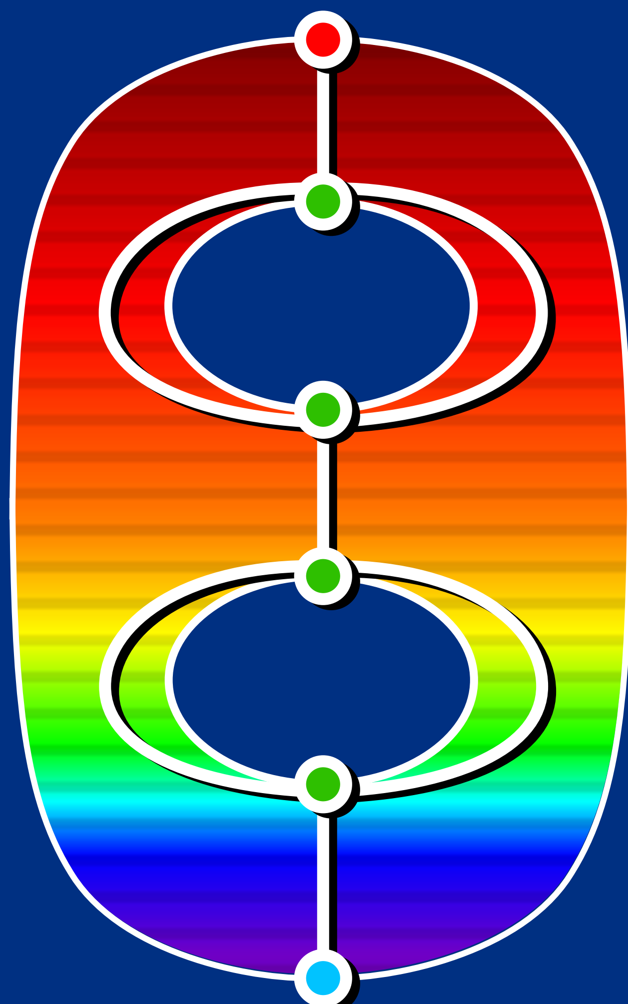
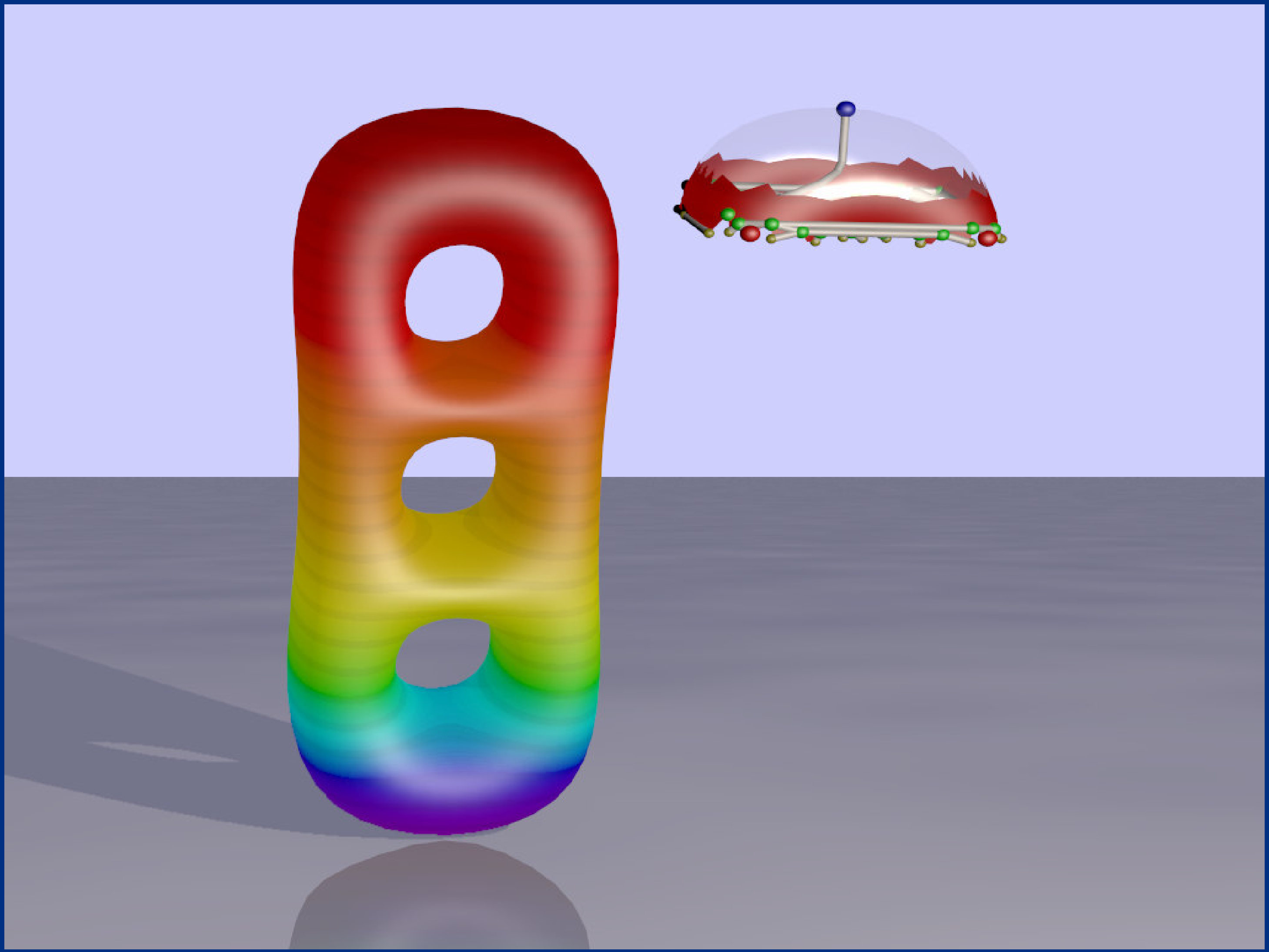


On-line Construction of the Reeb Graph for a Triple Torus Model



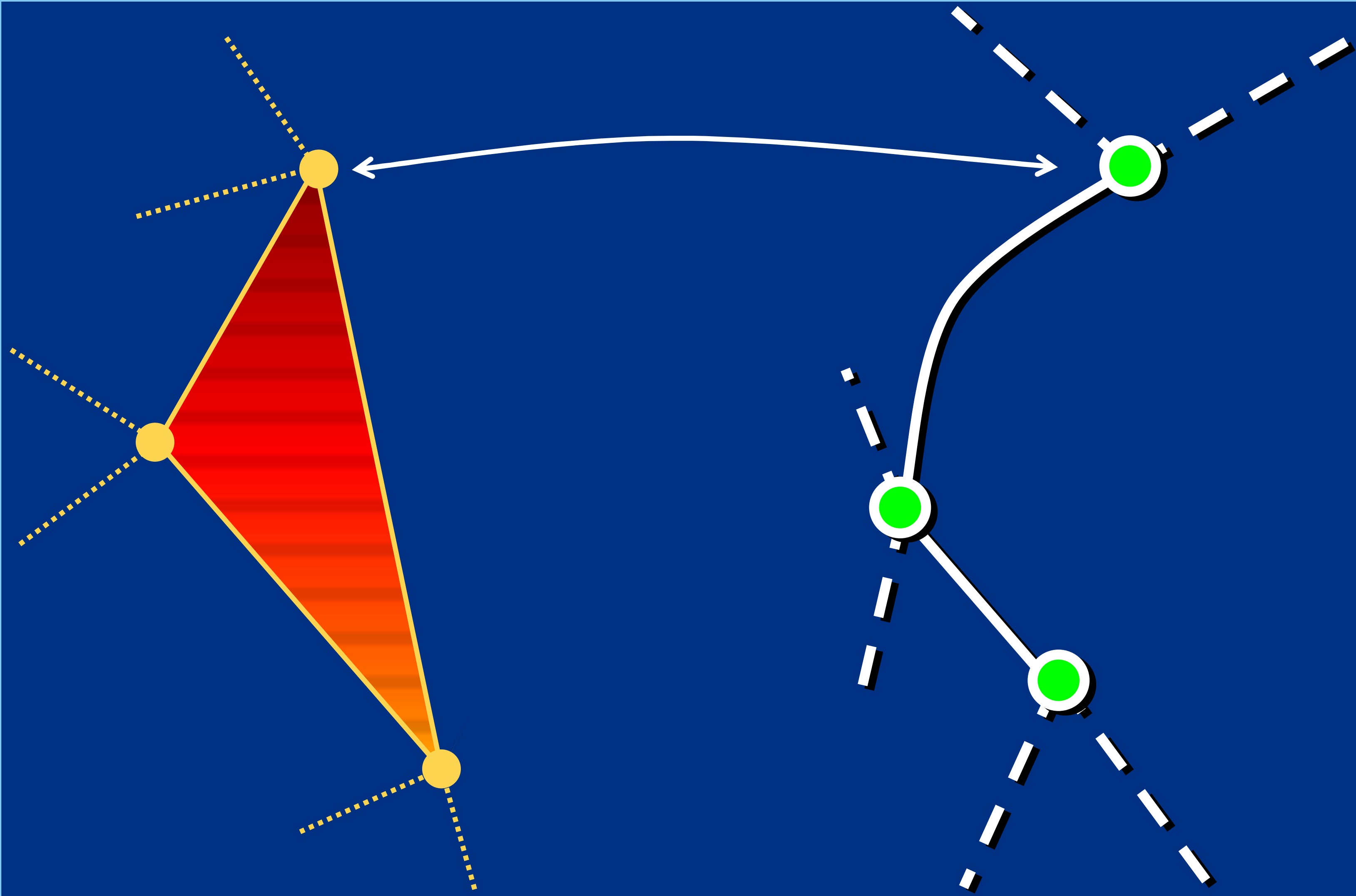
The Approach is Based on a Few Simple Assumptions on the Input

- Sequence of vertices, edges, and triangles.

- Indexed mesh: triangles/edges defined as references to vertices.

- Finalization: each vertex knows when it is referenced for the last time (if needed a pass can collect this information easily and fast).

Data Structure for Incremental Update of the Reeb Graph



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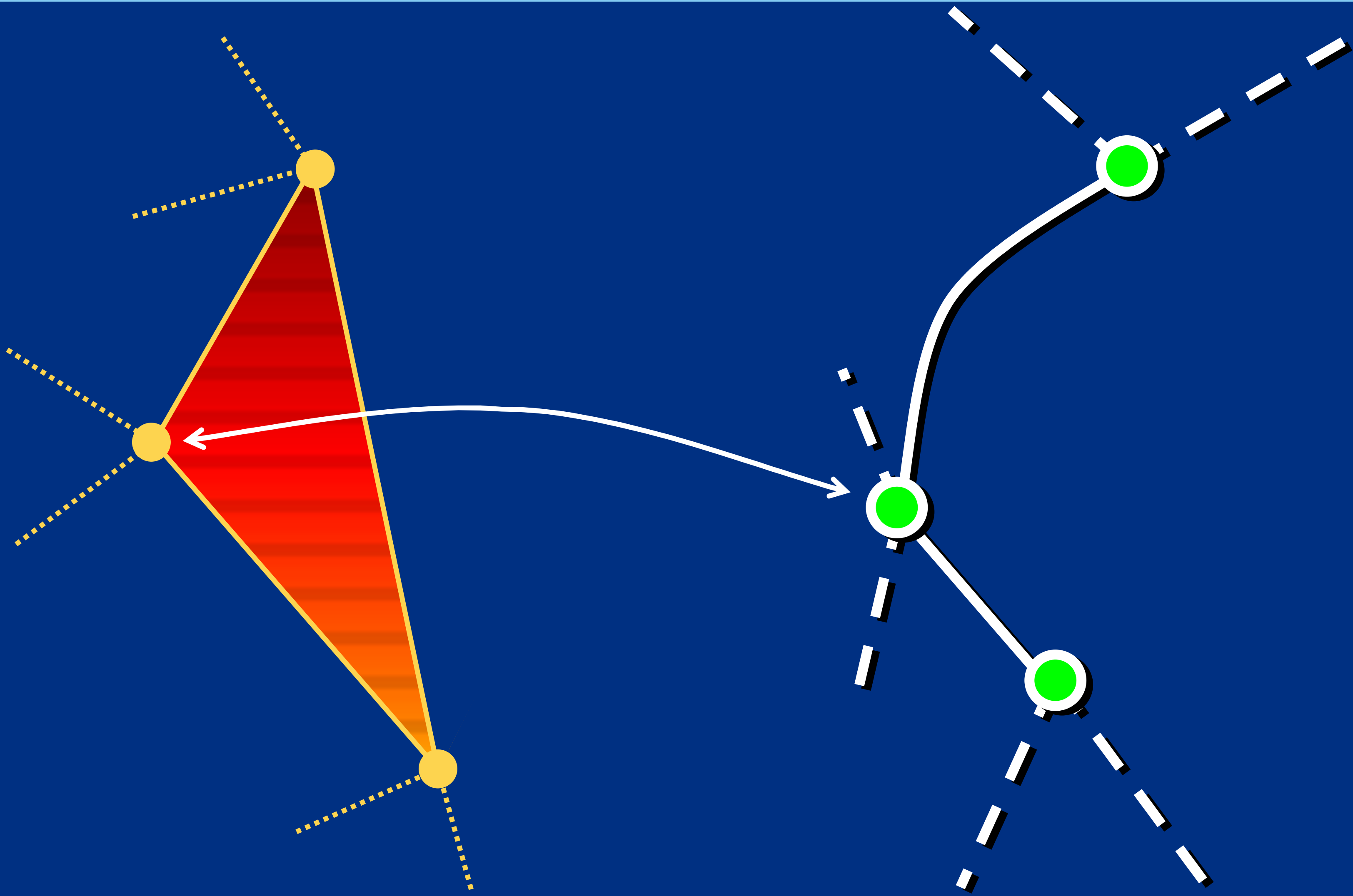

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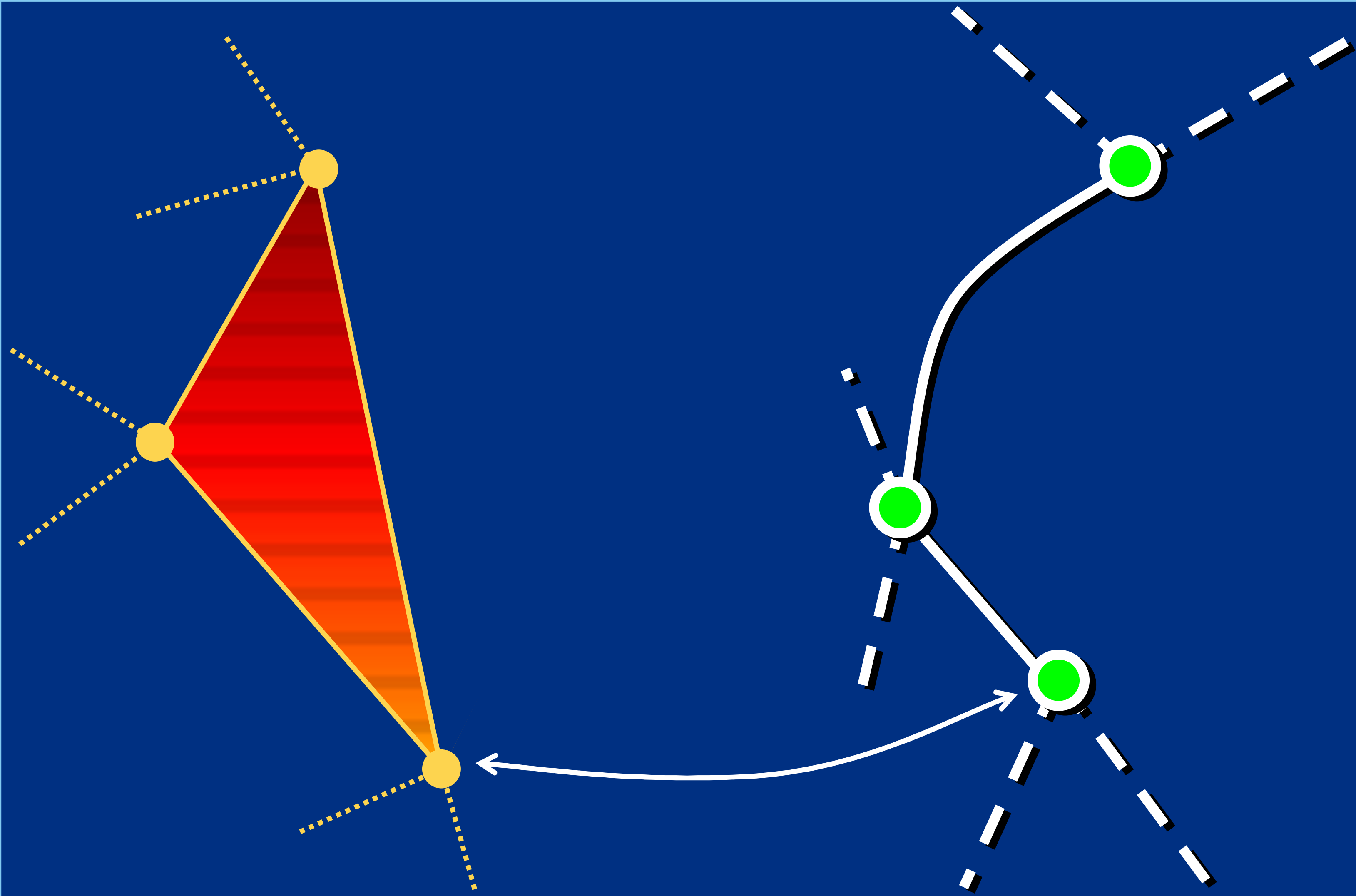

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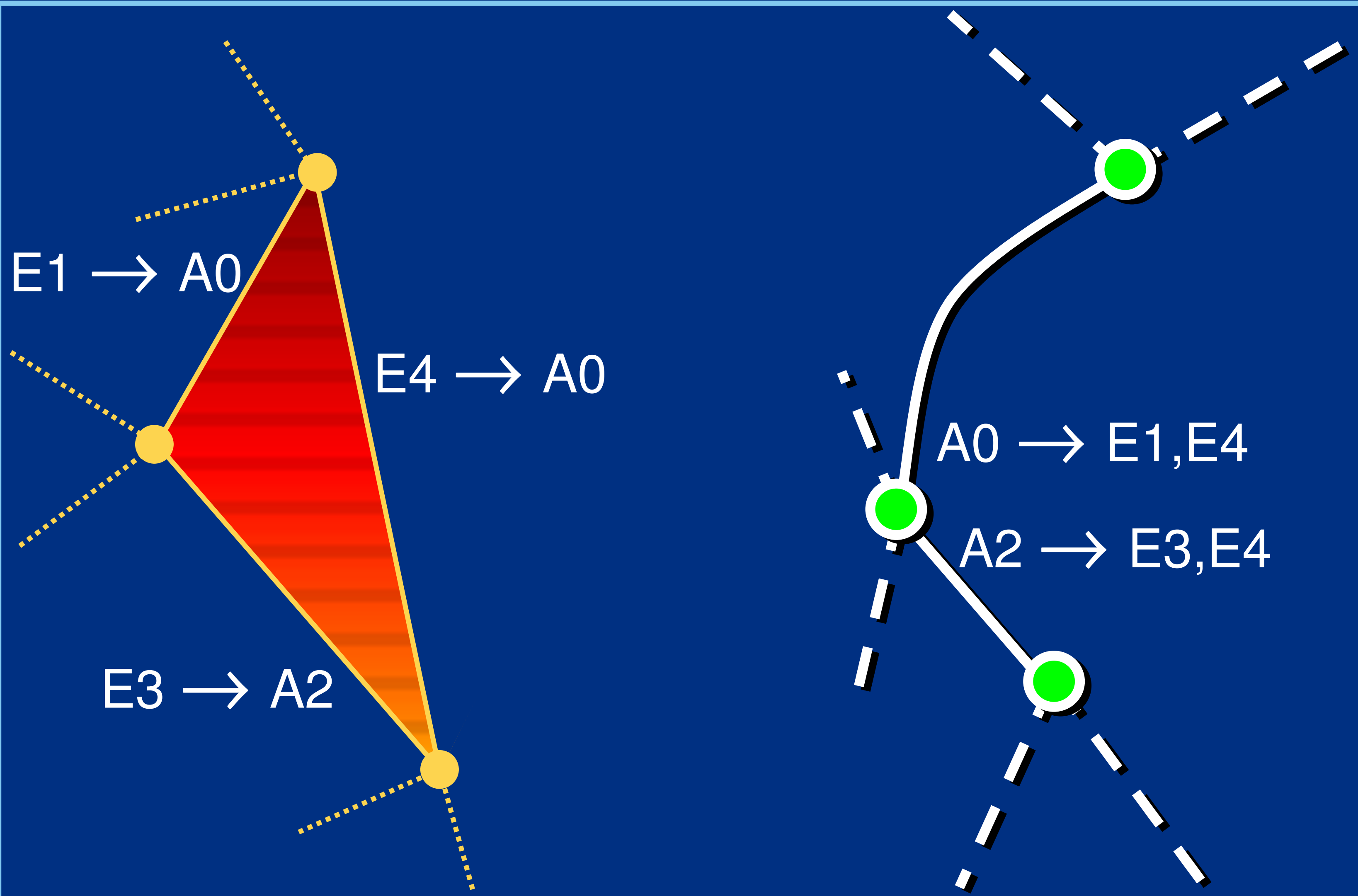

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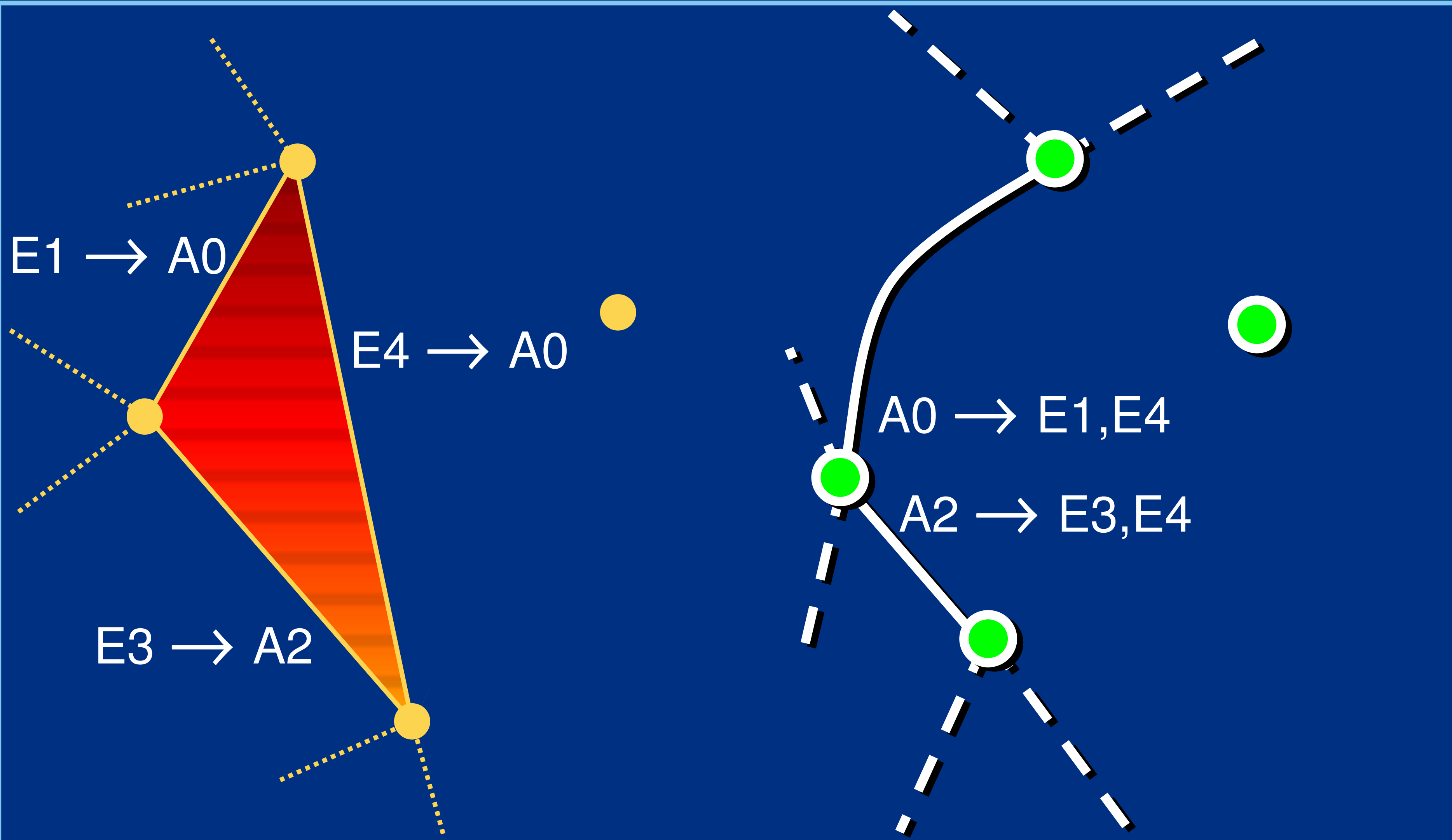

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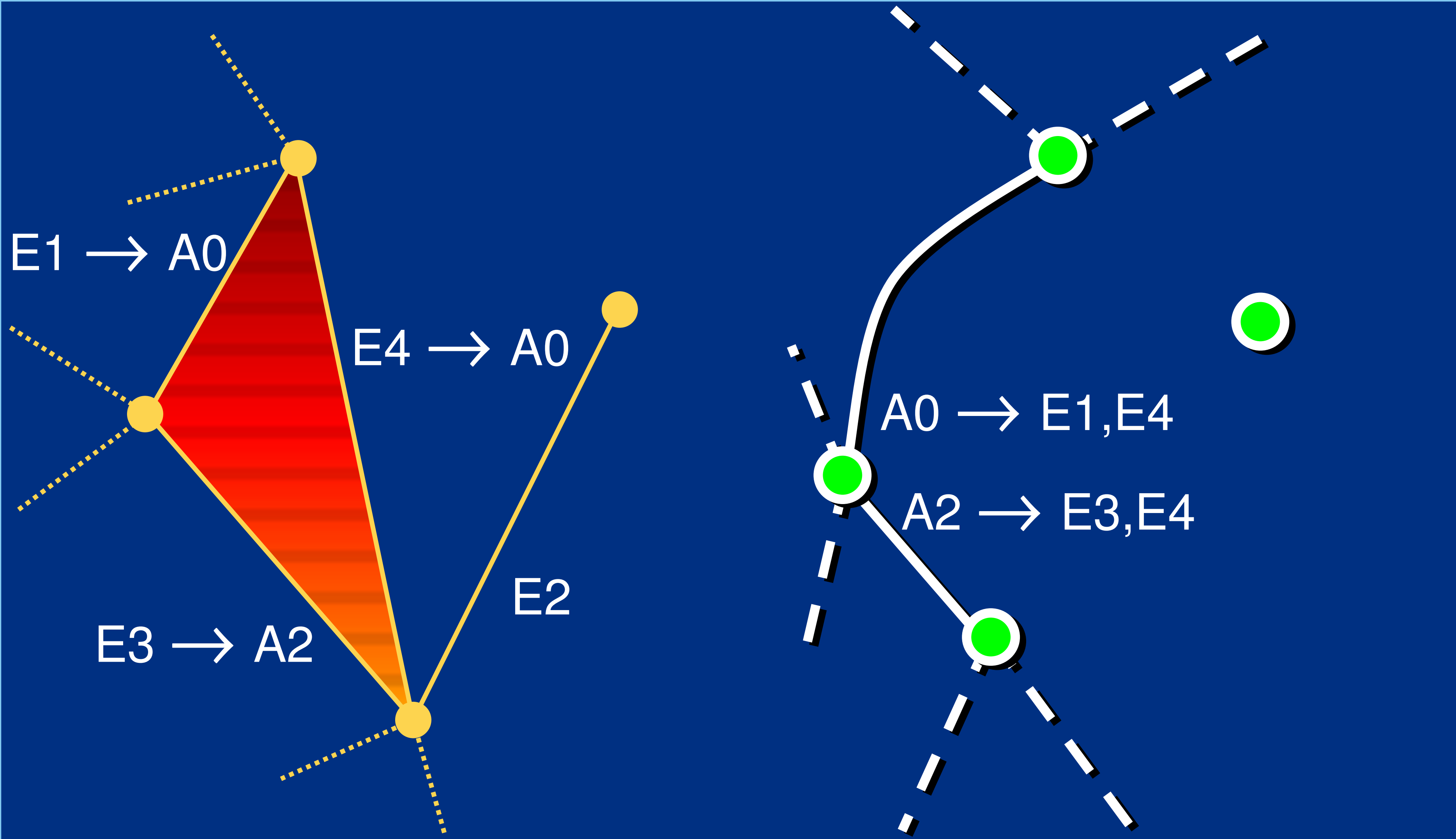

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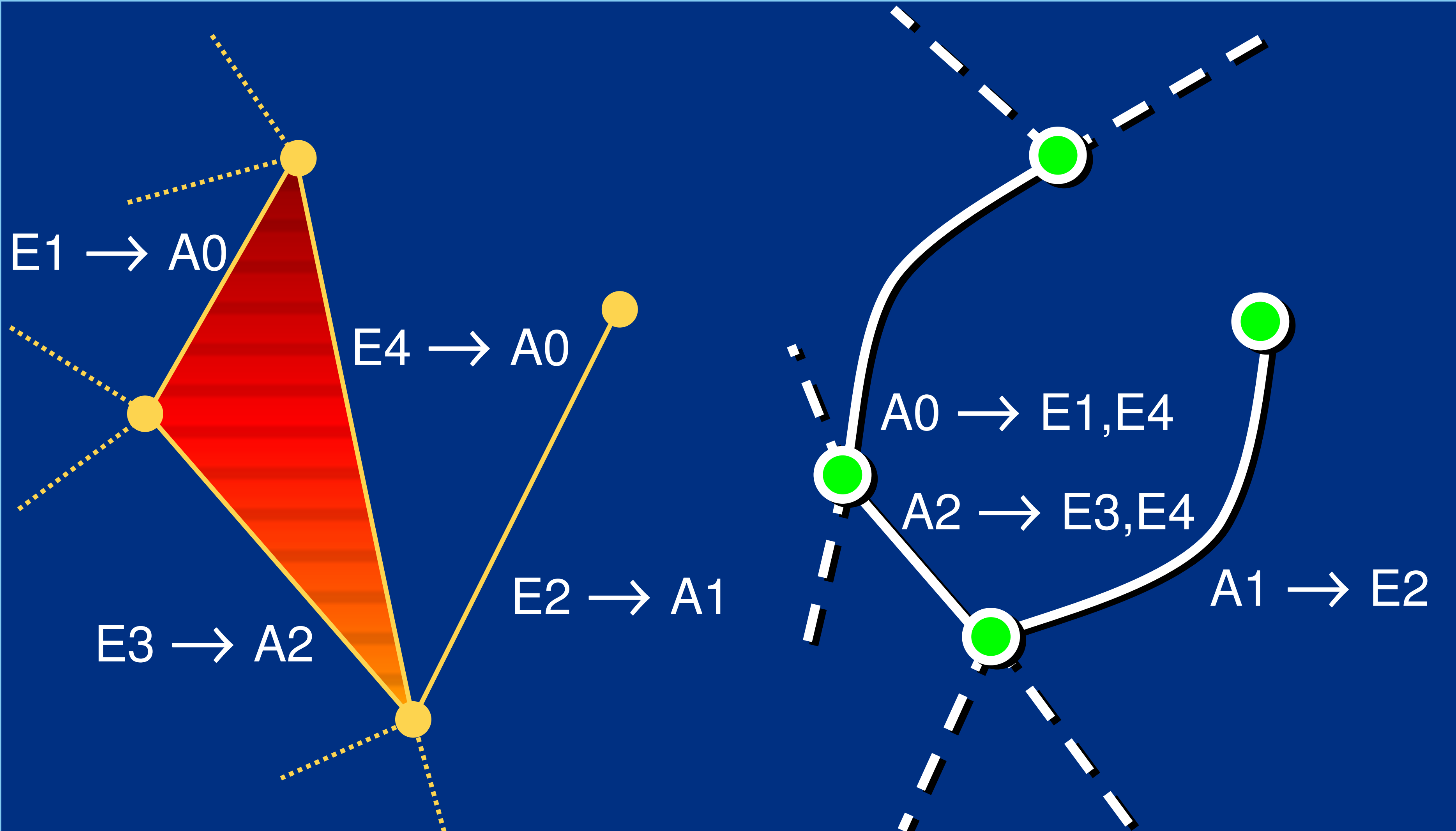

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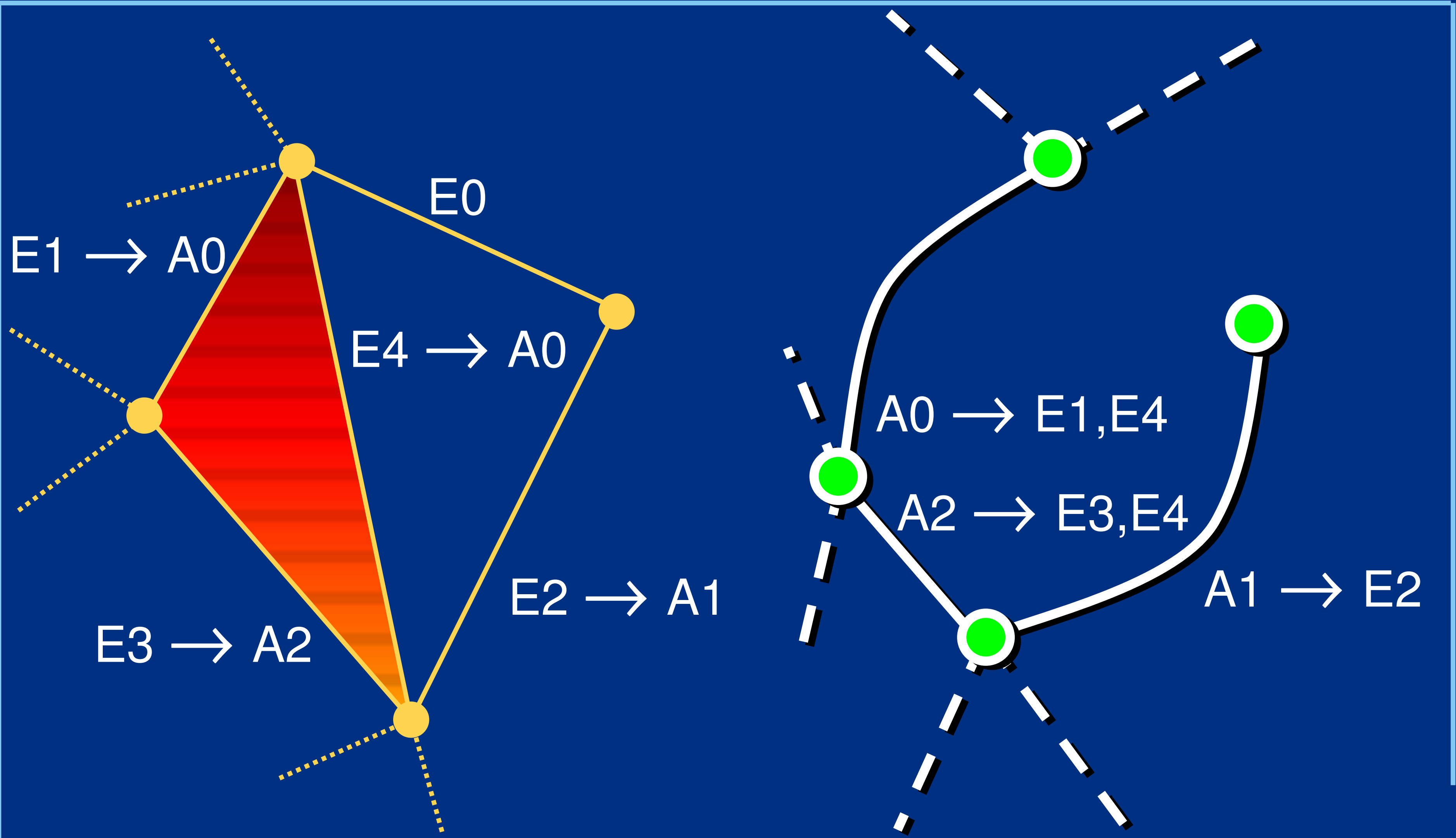

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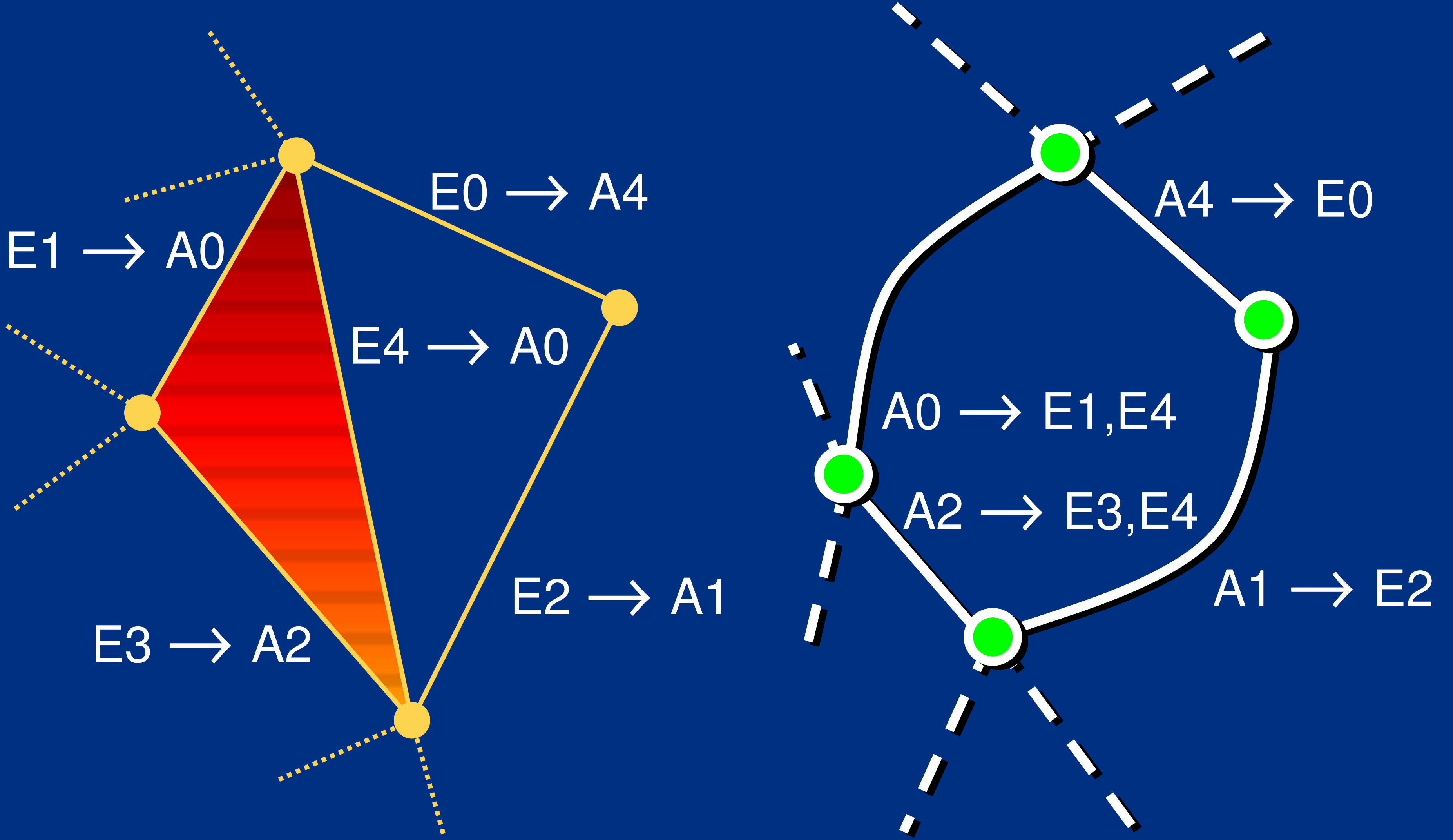

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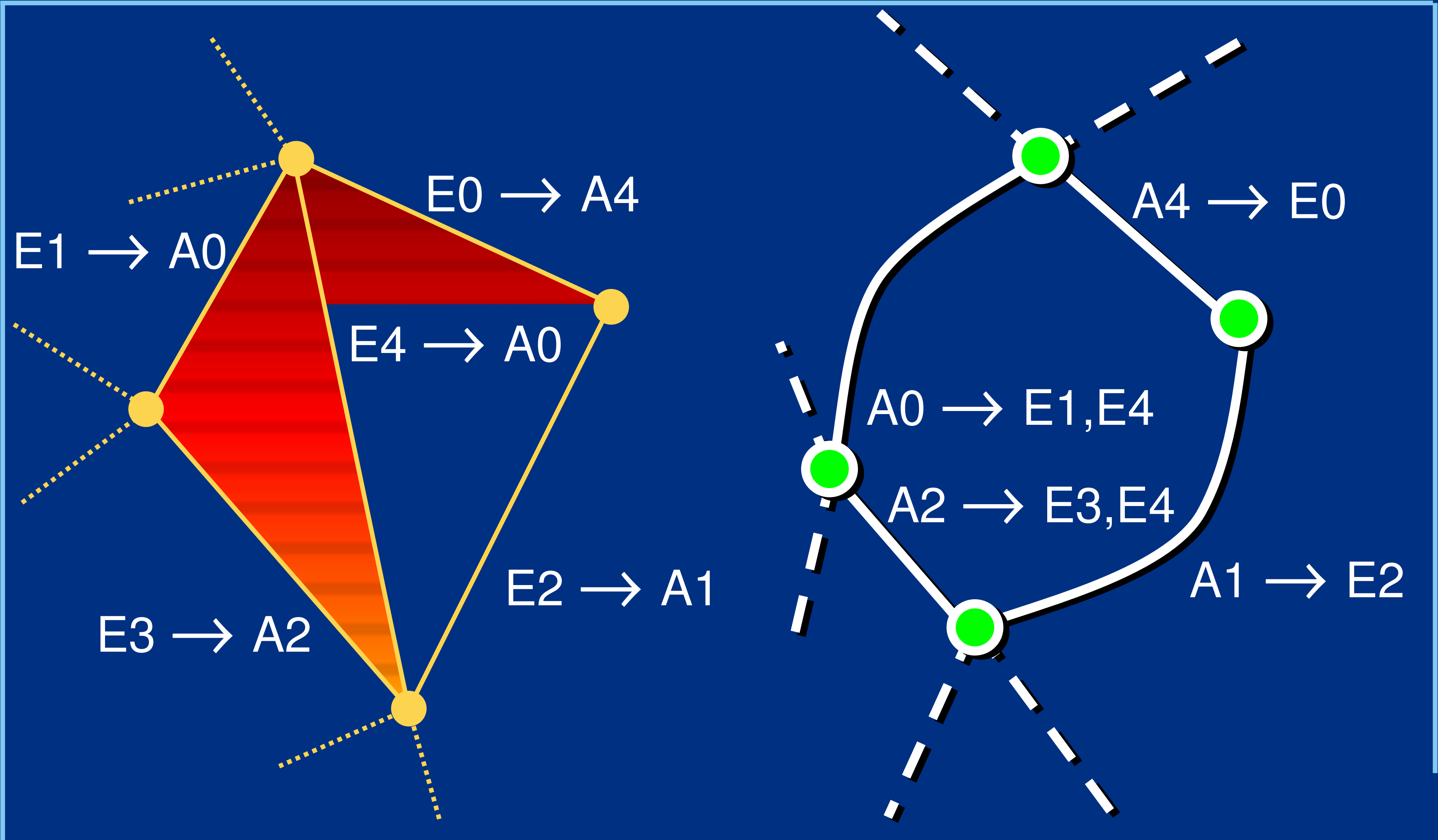

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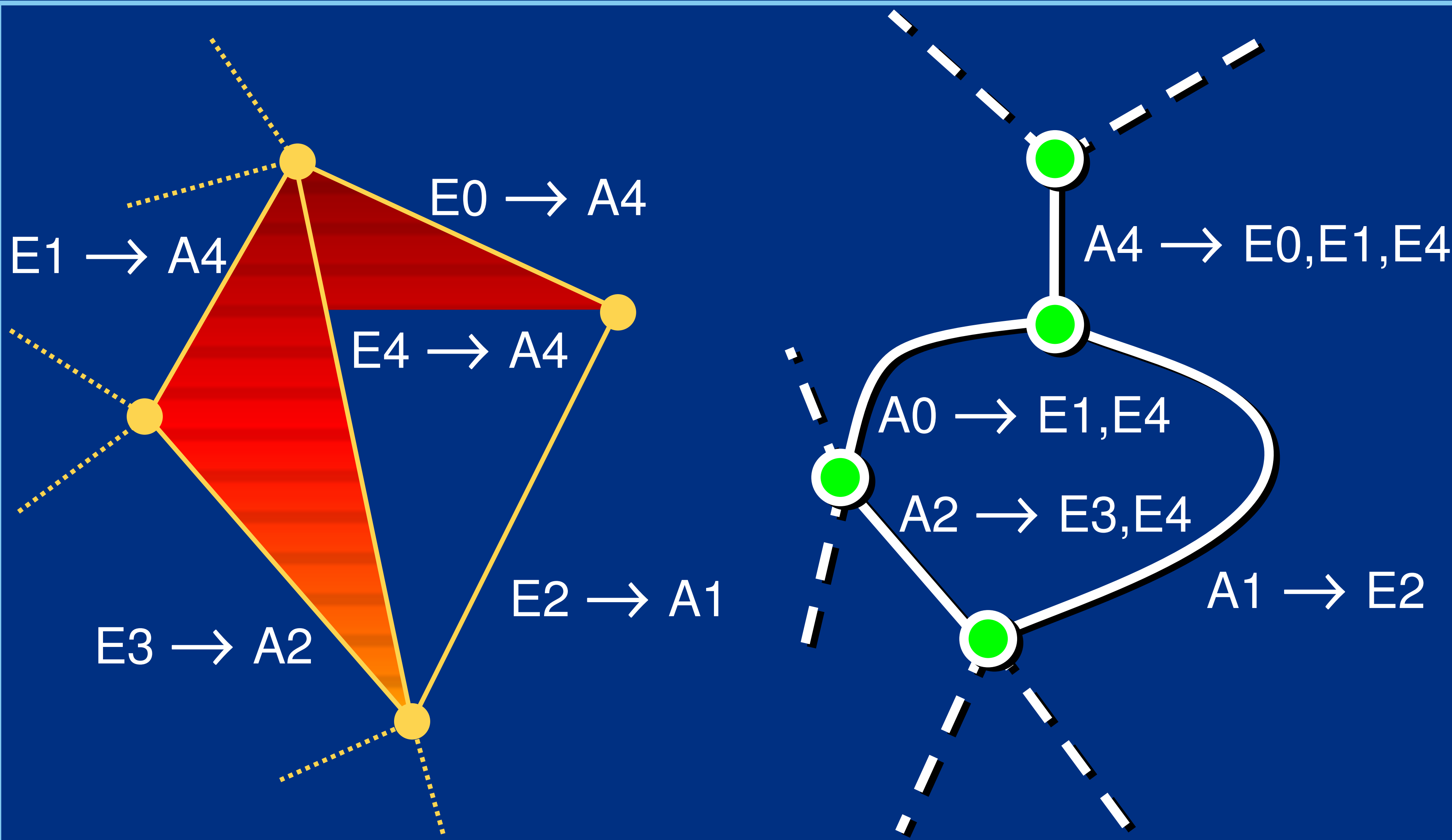

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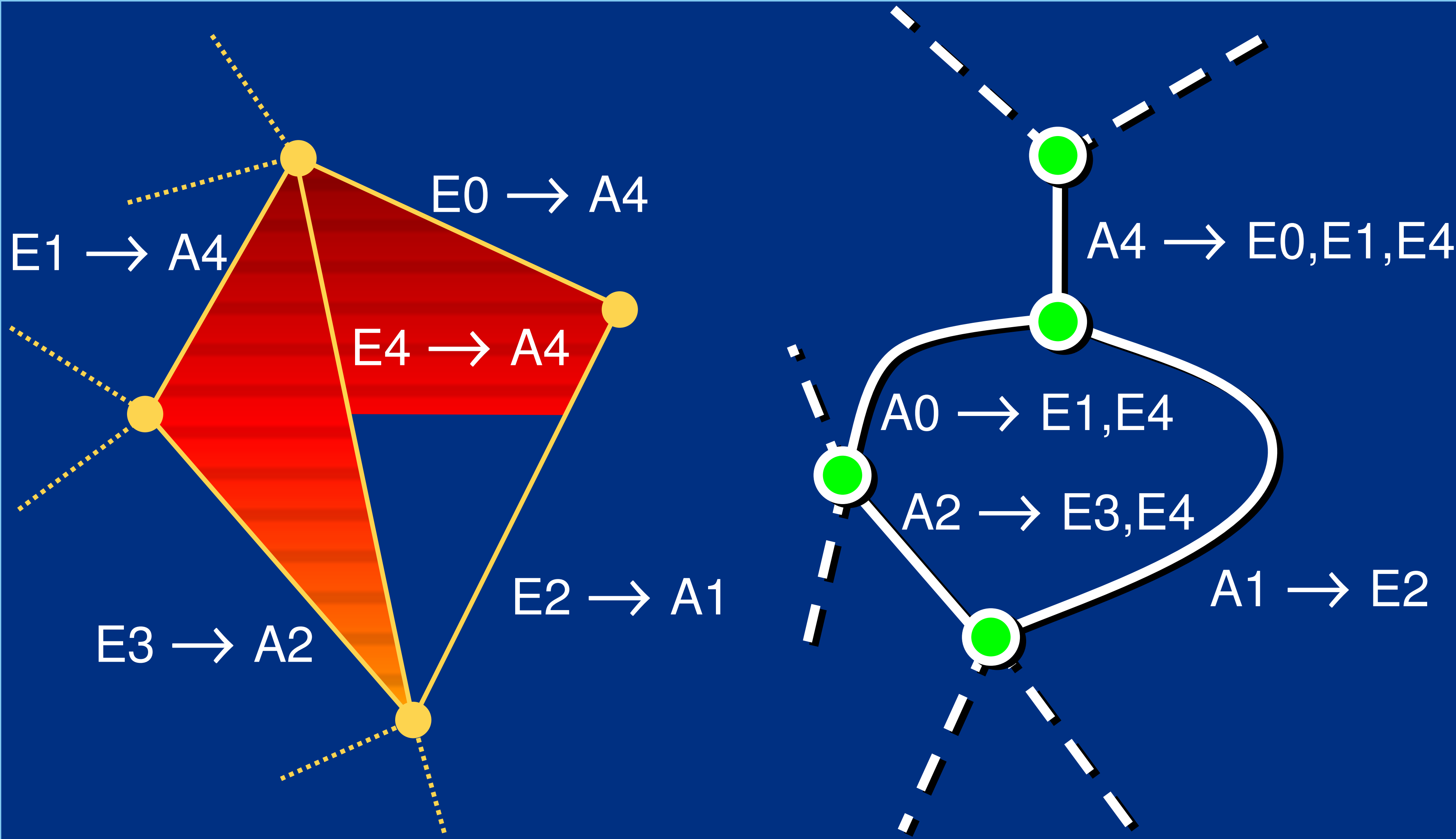

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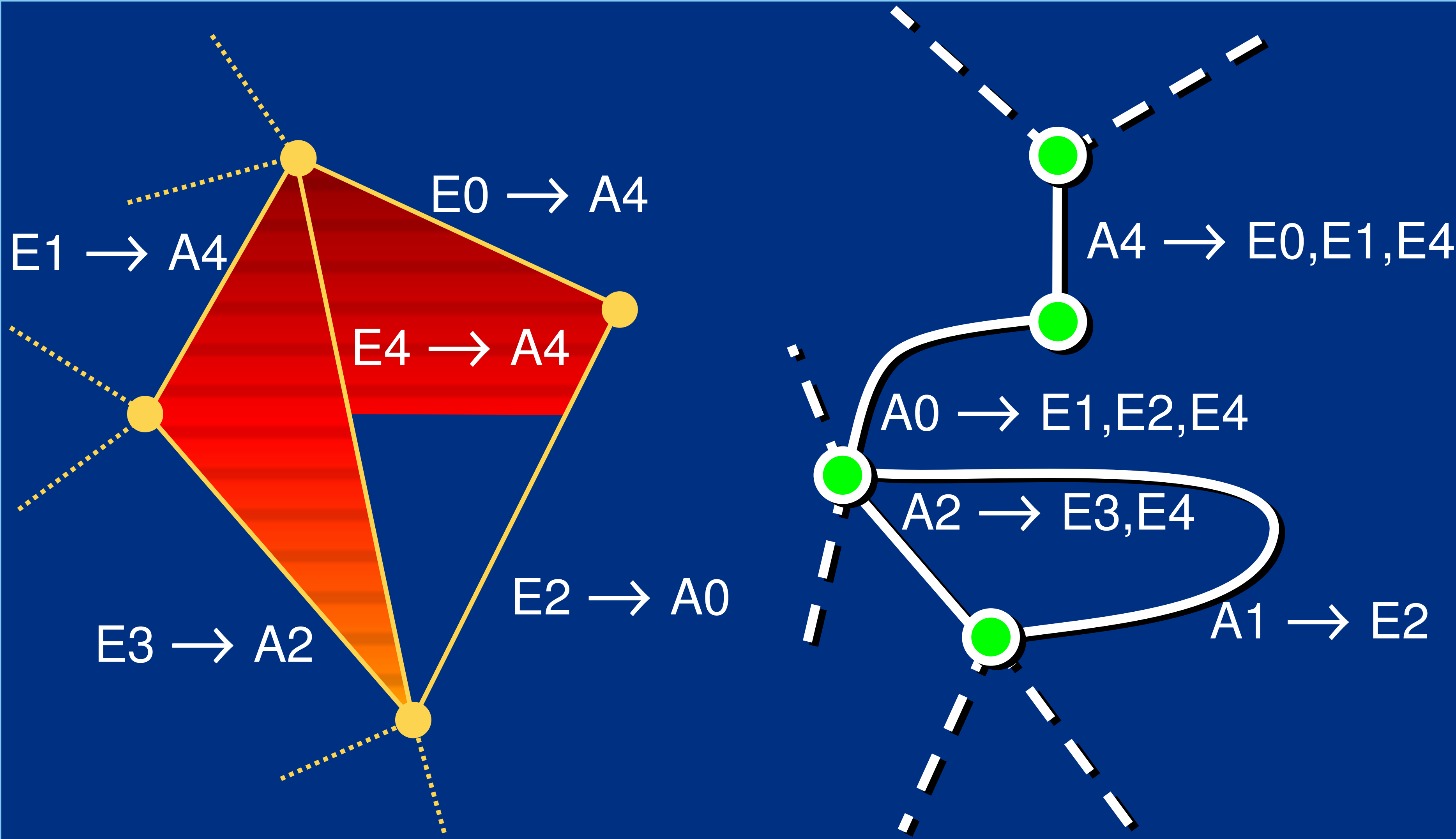

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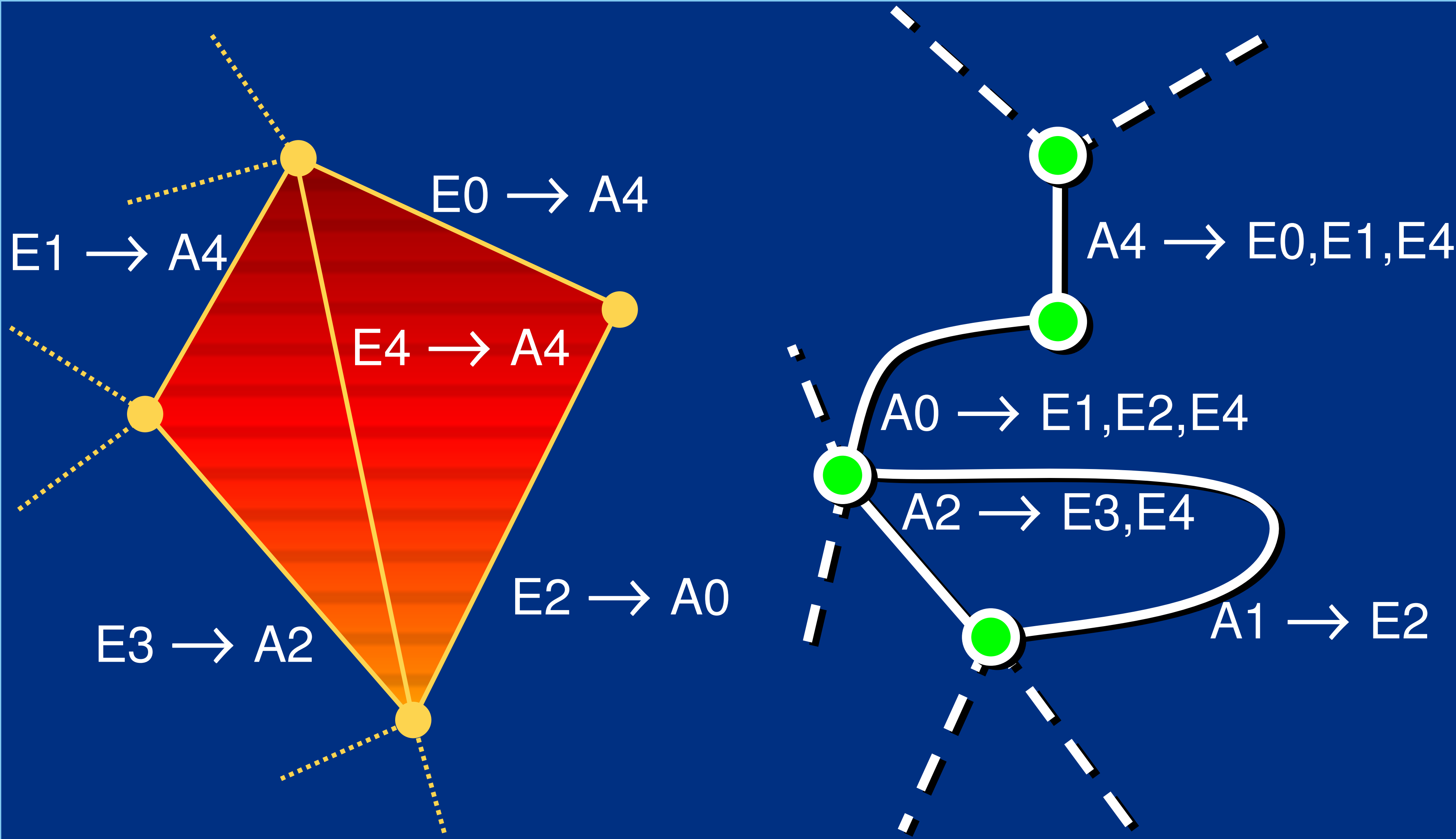

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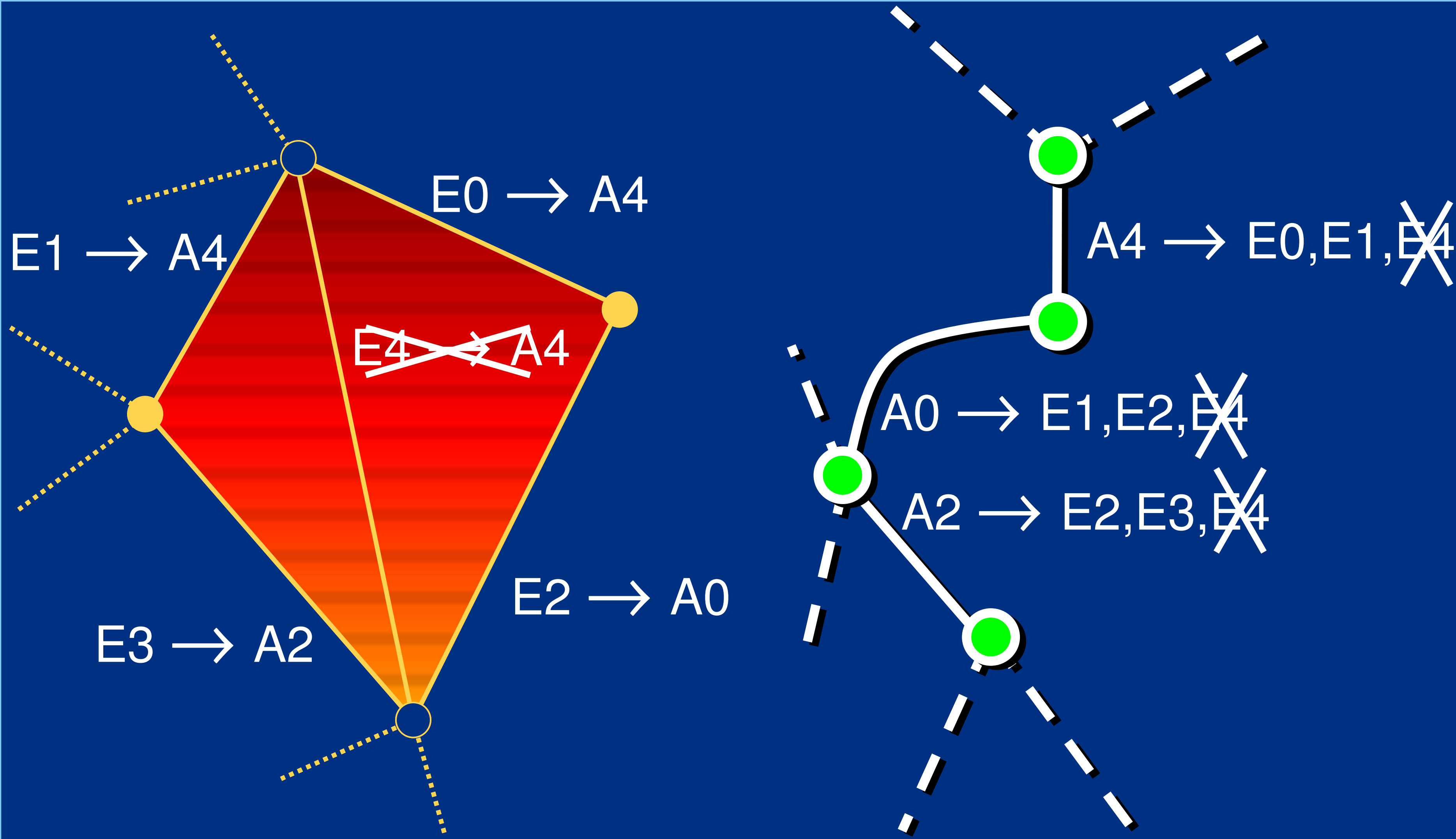

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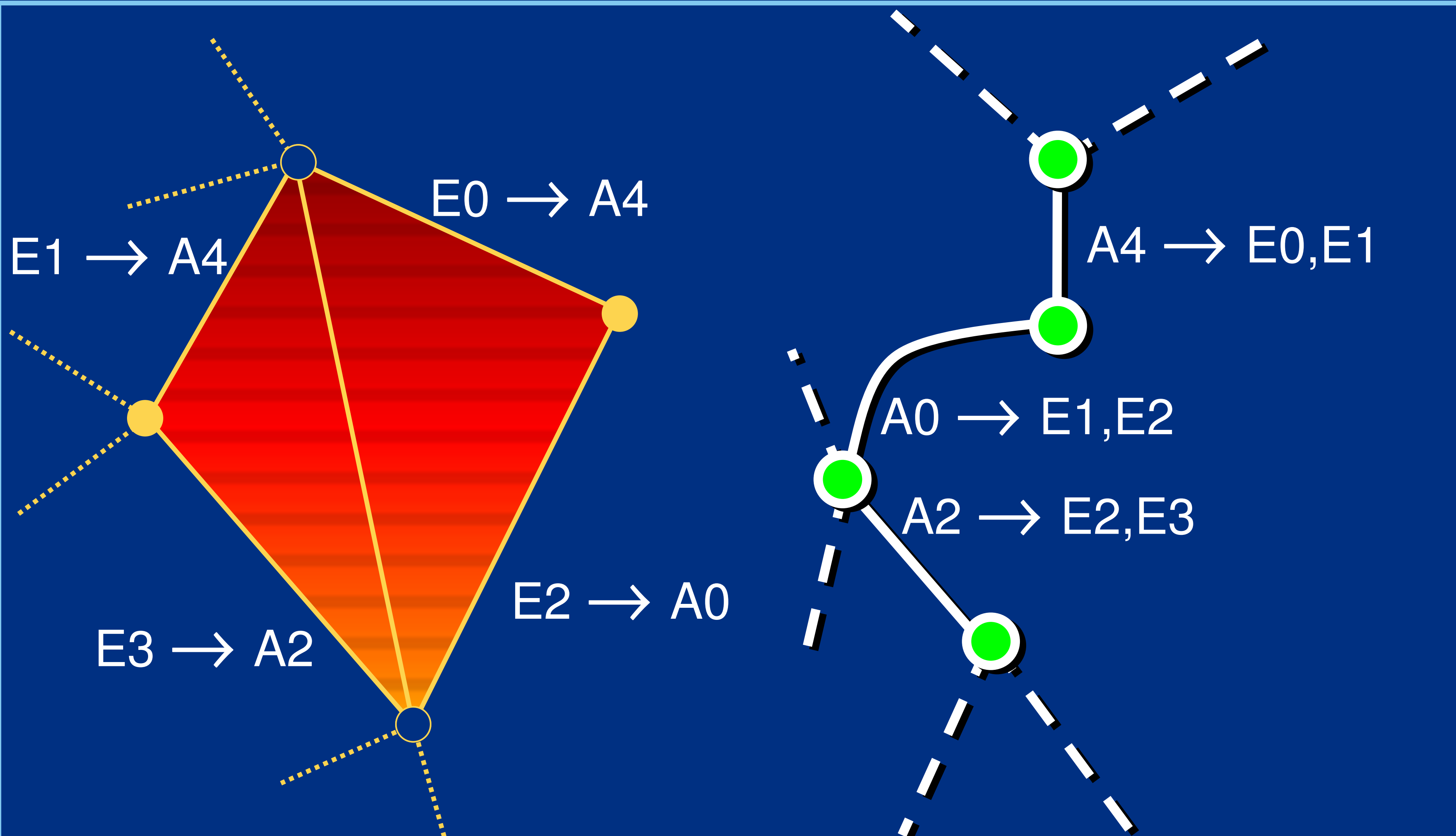

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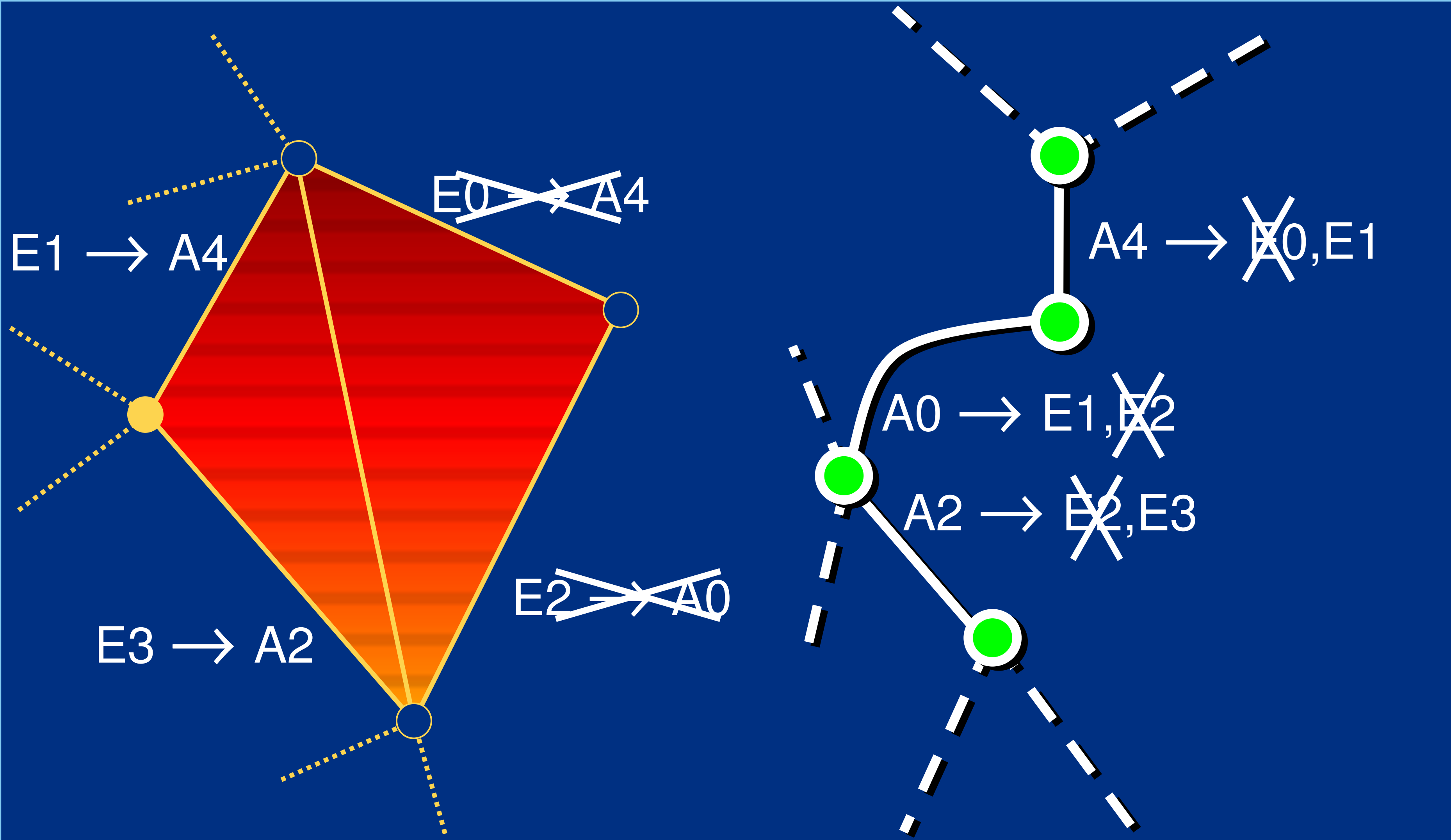

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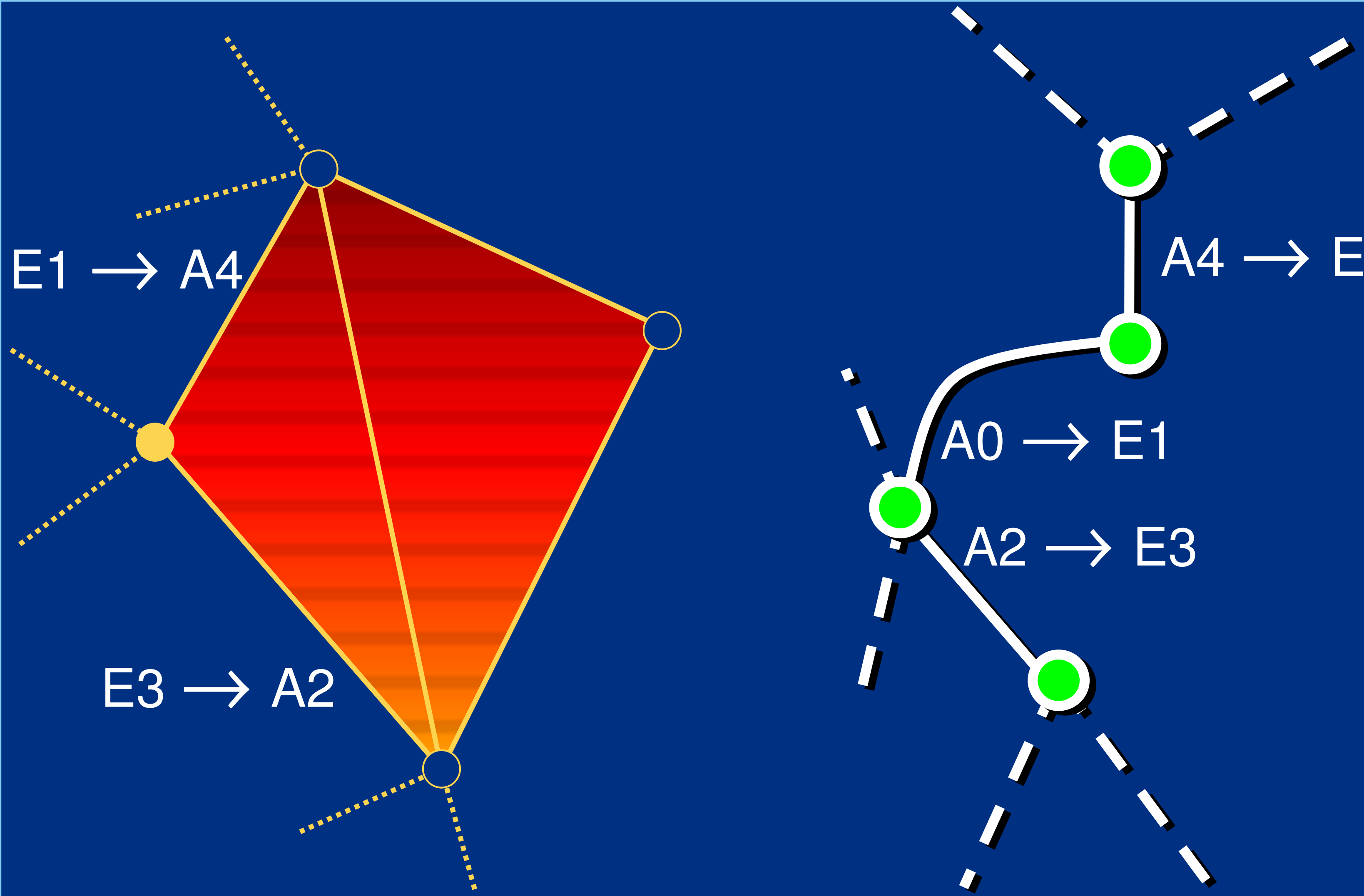

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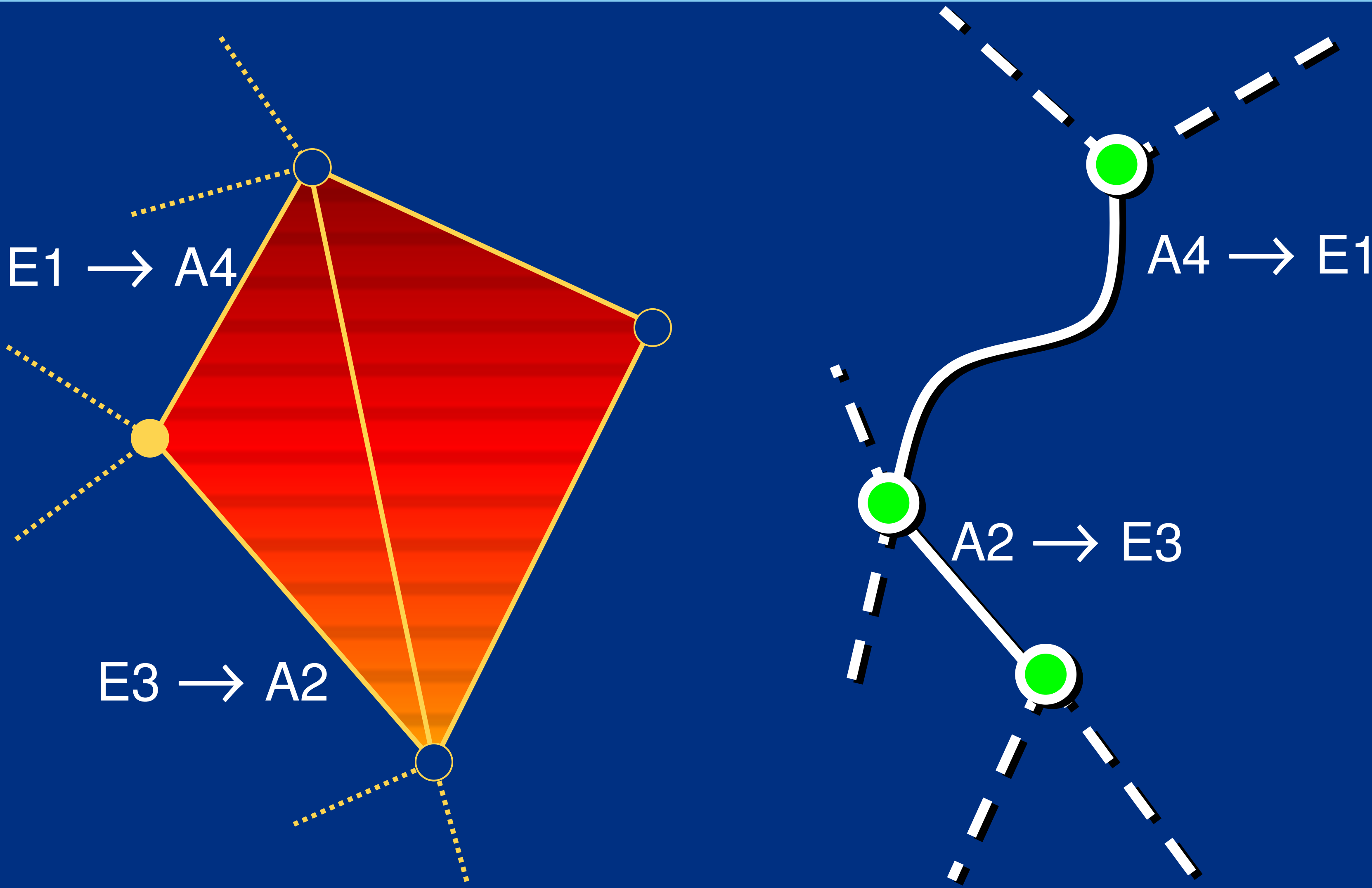
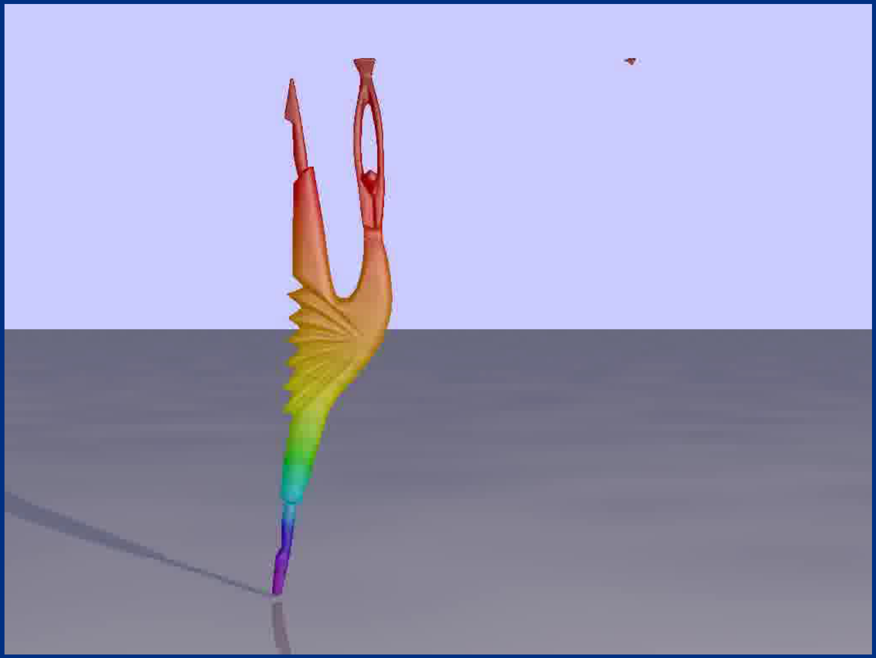


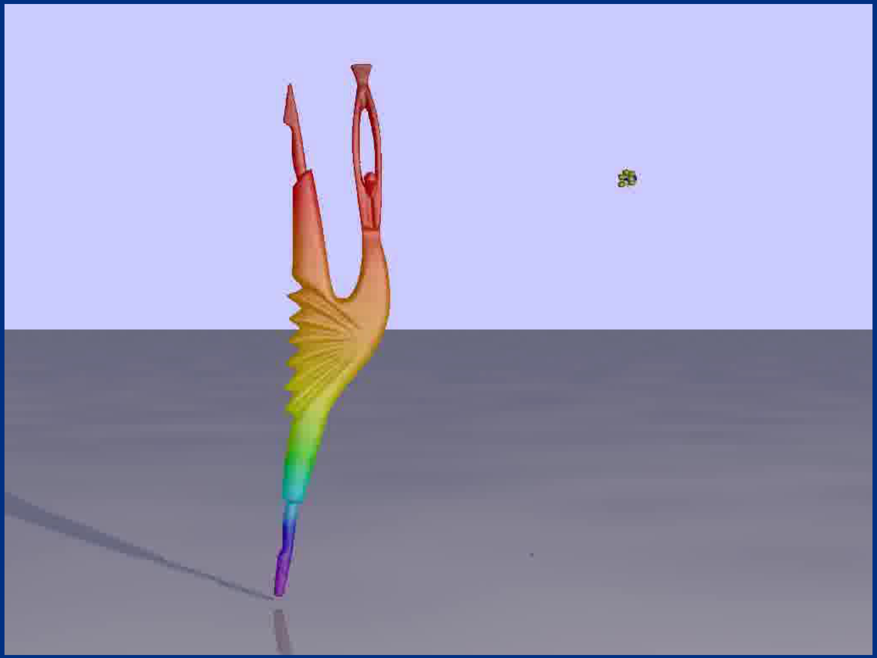
In Out of Core Mode Unnecessary Elements Are Removed from Memory

- From input mesh:
  - Never store triangles.
  - Remove finalized vertices.
  - Remove edges with finalized vertices.
- From Reeb graph:
  - Retire arcs without edge reference.
  - Retire nodes without edge reference and adjacent arcs.

The Input Triangles Do Not Need to Be in Any Particular Order

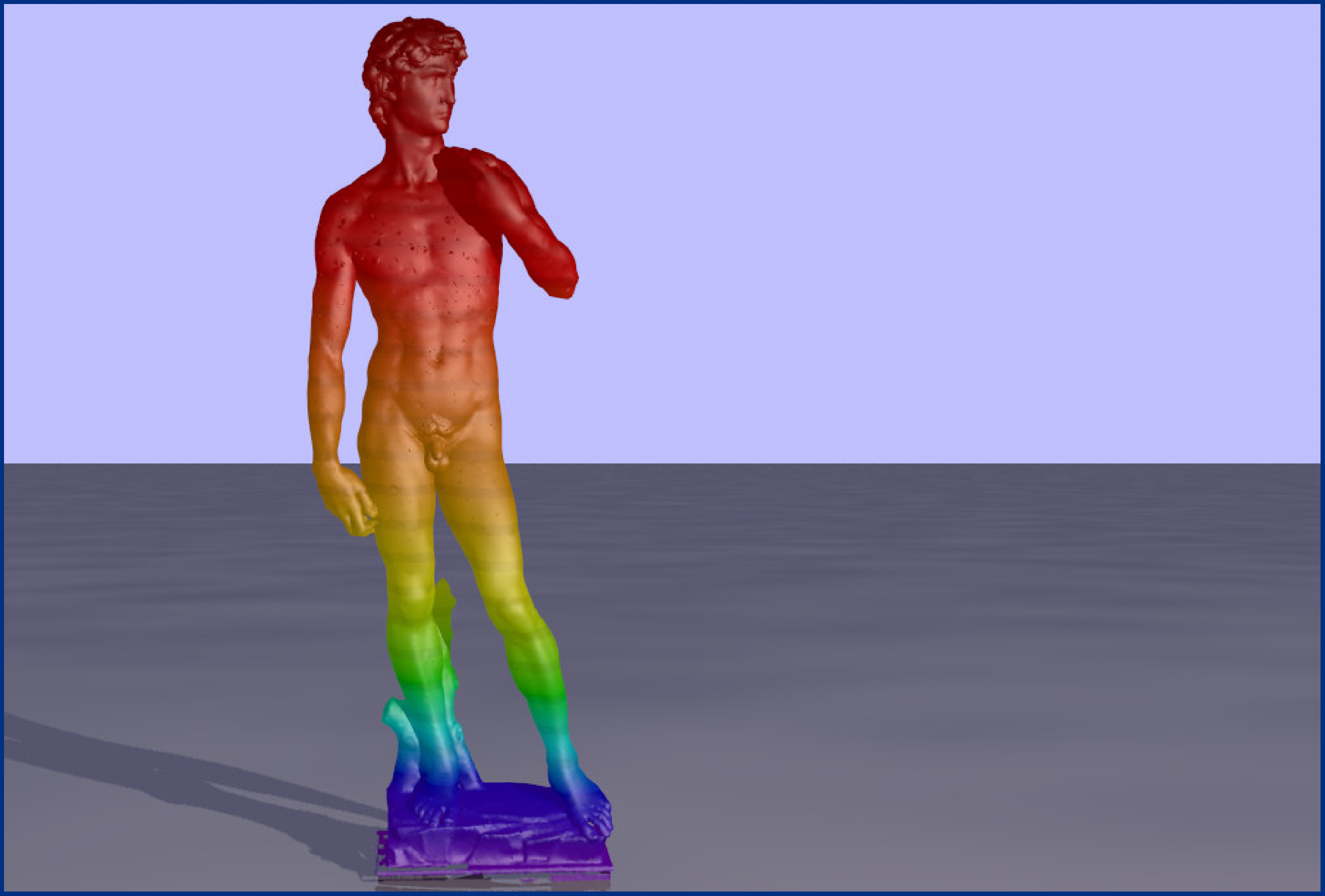
- Dancer model with 50K triangles.





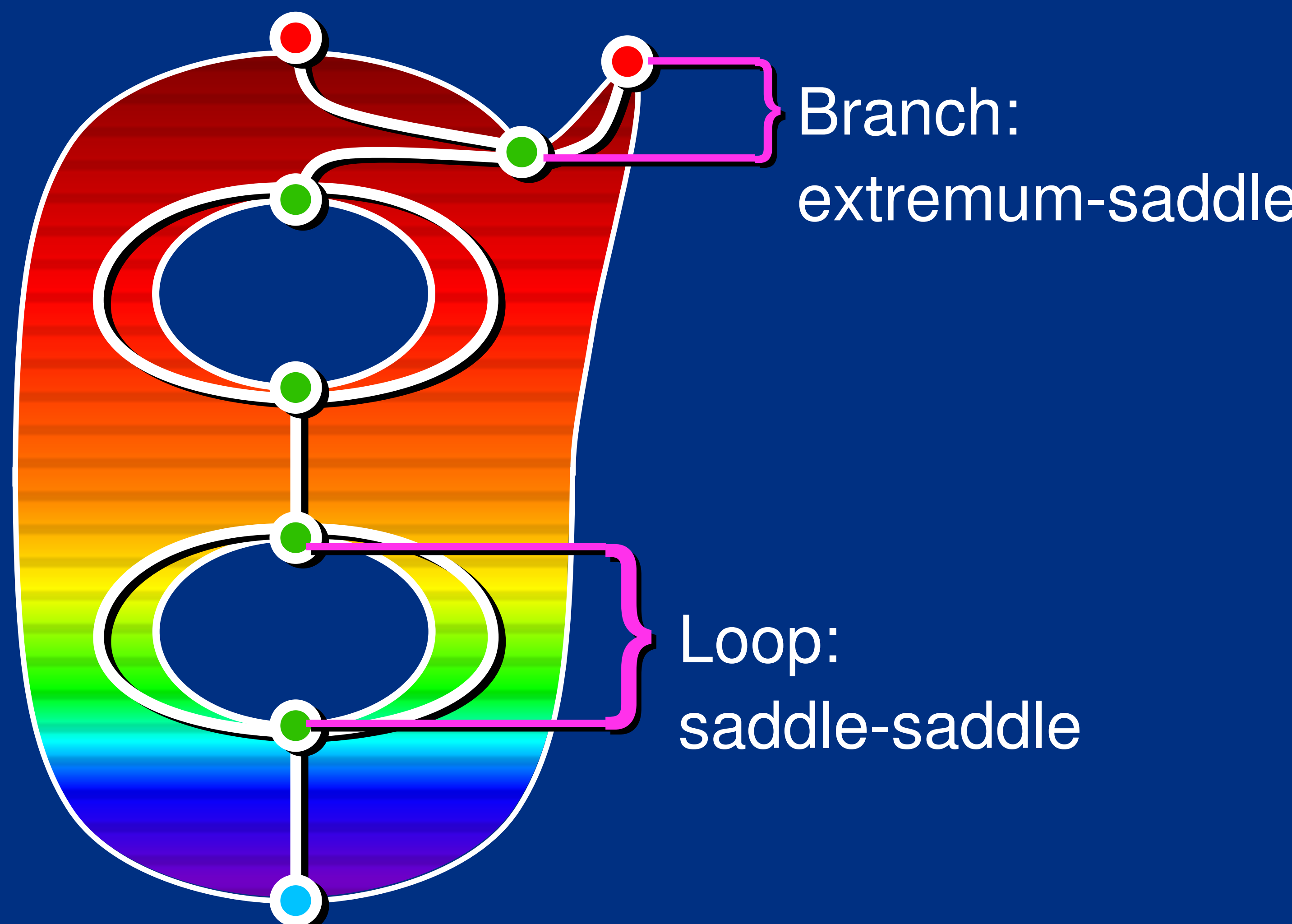
- Sorted by Z coordinate: 0.11s, 1.8MB.
- Original mesh: 0.2s, 1.8MB.

We Exploit the Locality of an Input Mesh in Cache Friendly Format



David model: 56Mt, 108s, 2.1MB

The Reeb Graph Can Be Simplified by Removing Branches and Loops



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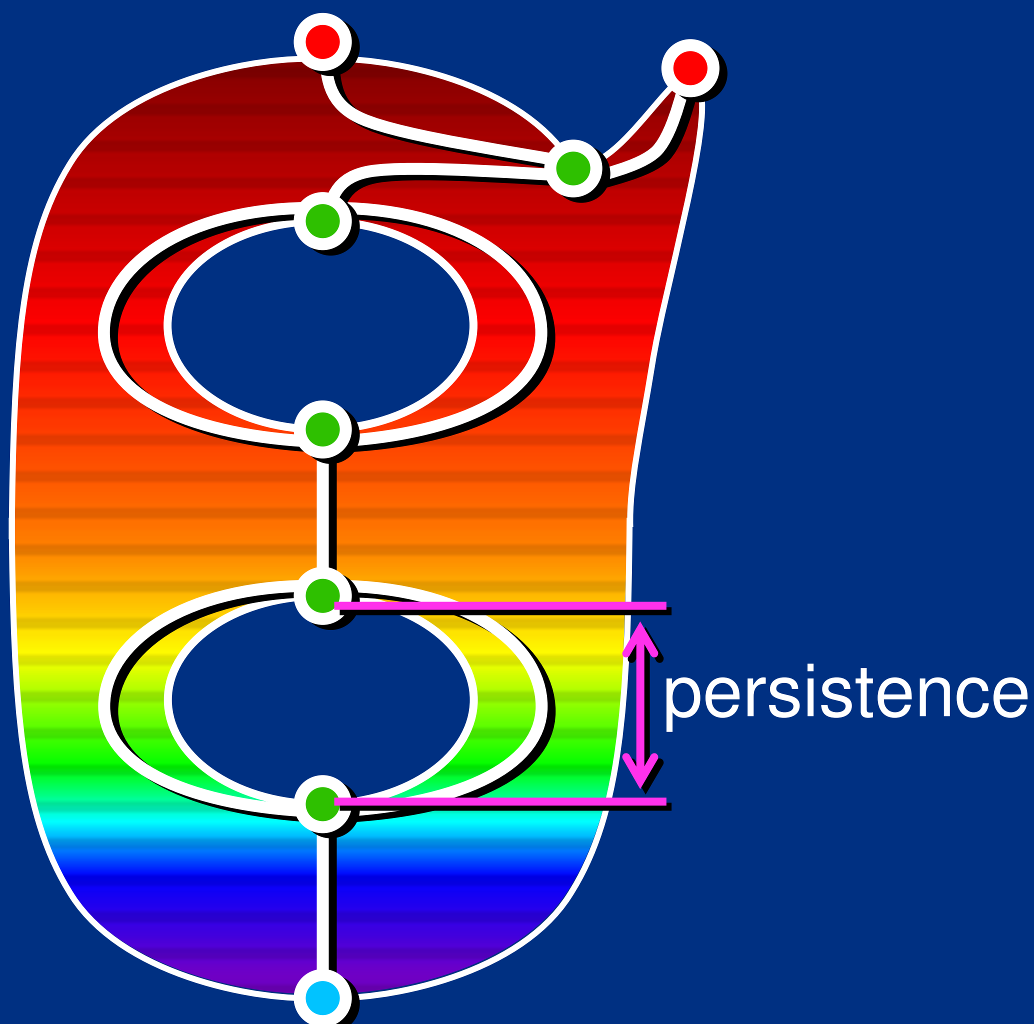
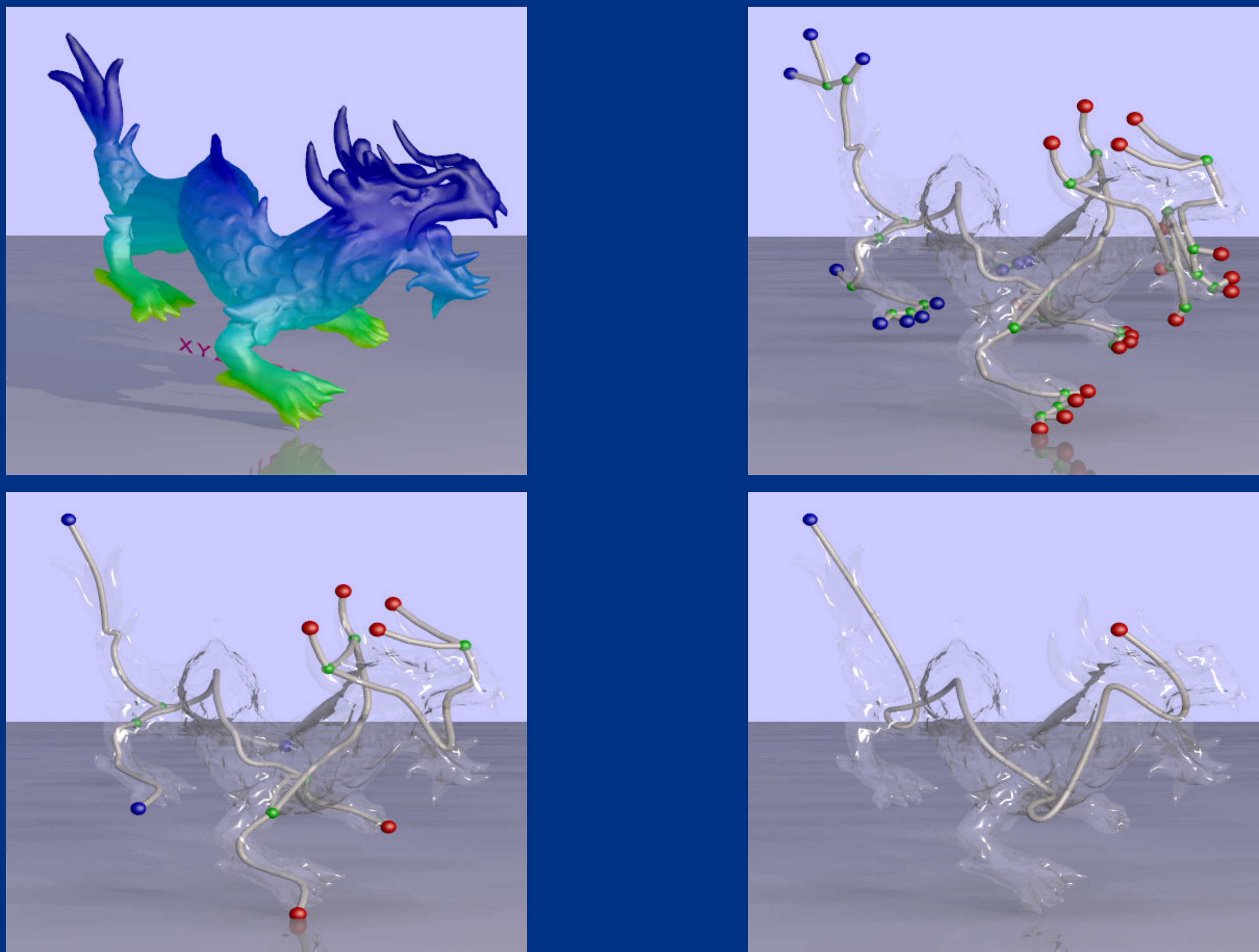
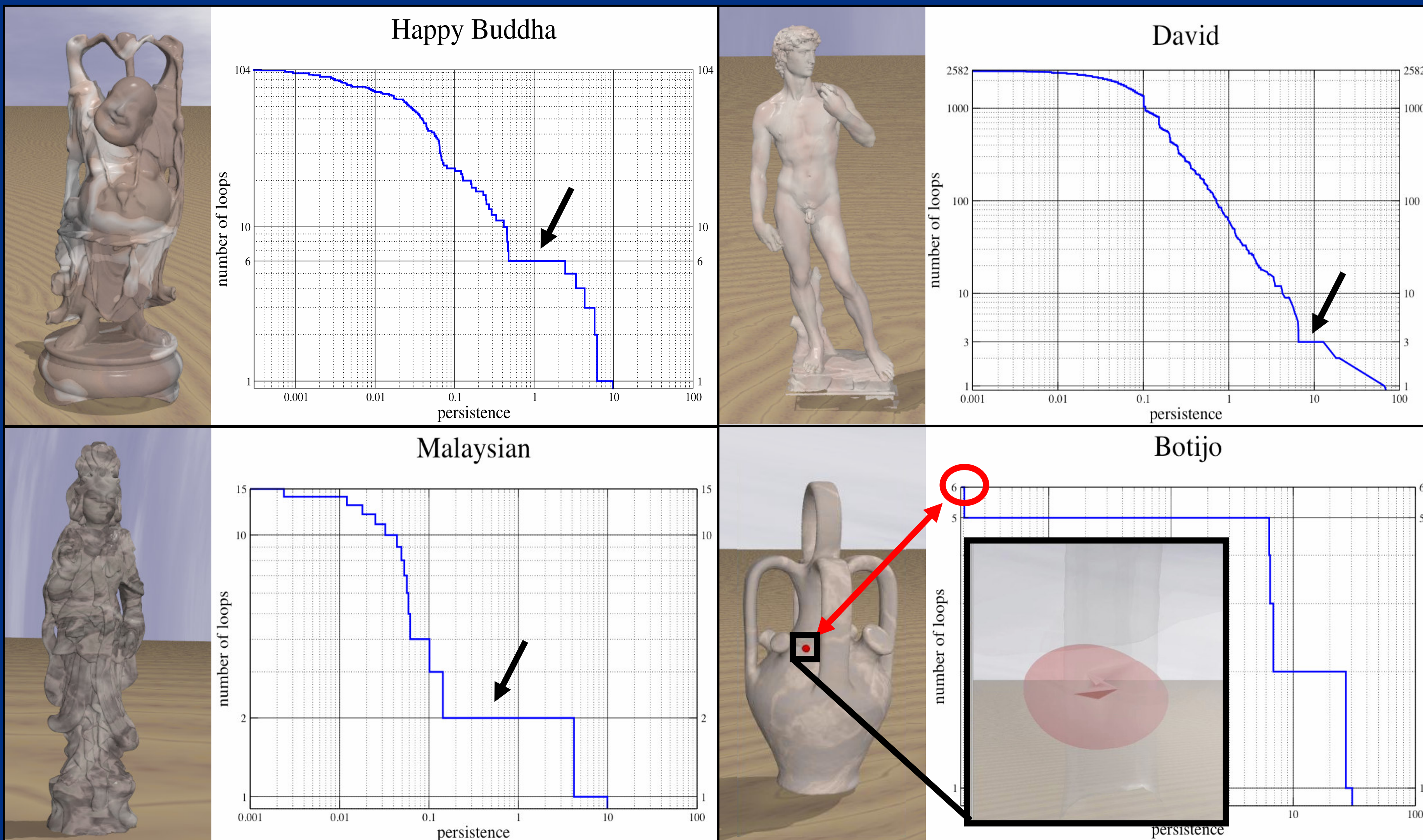


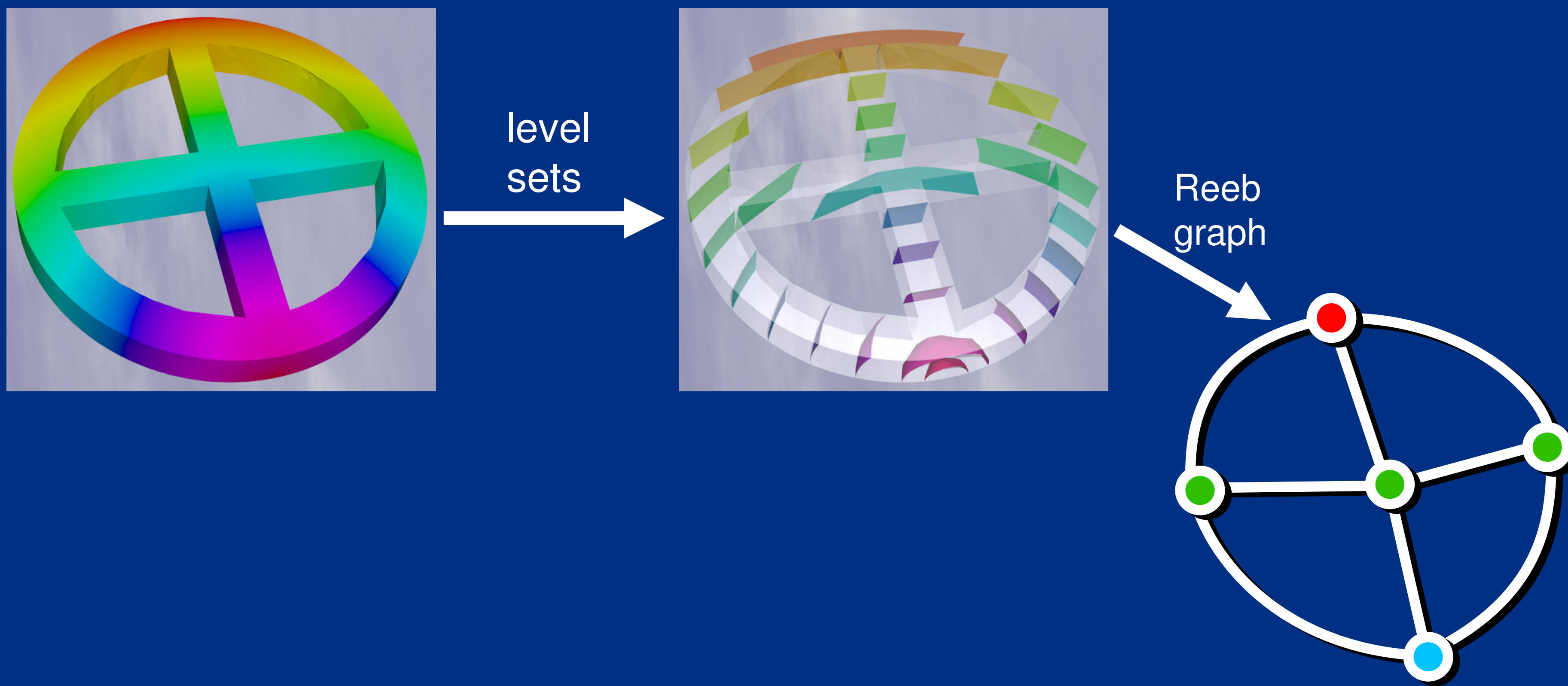
The User Can Choose the Level of Resolution for the Reeb Graph



The Simplification Can Be Used to Develop Shape Signatures



The Scheme Computes Reeb Graphs for Meshes of Any Dimension



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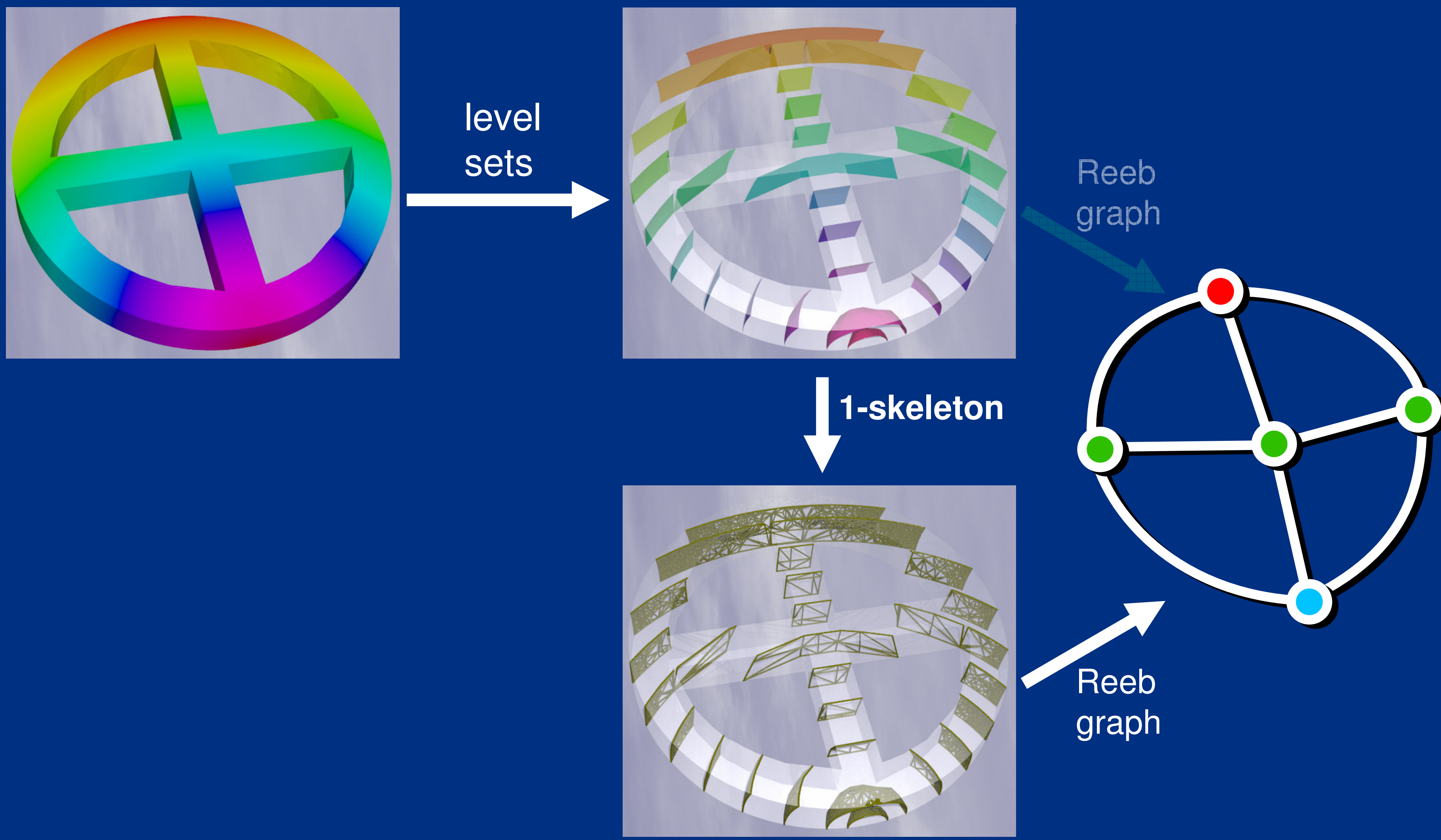

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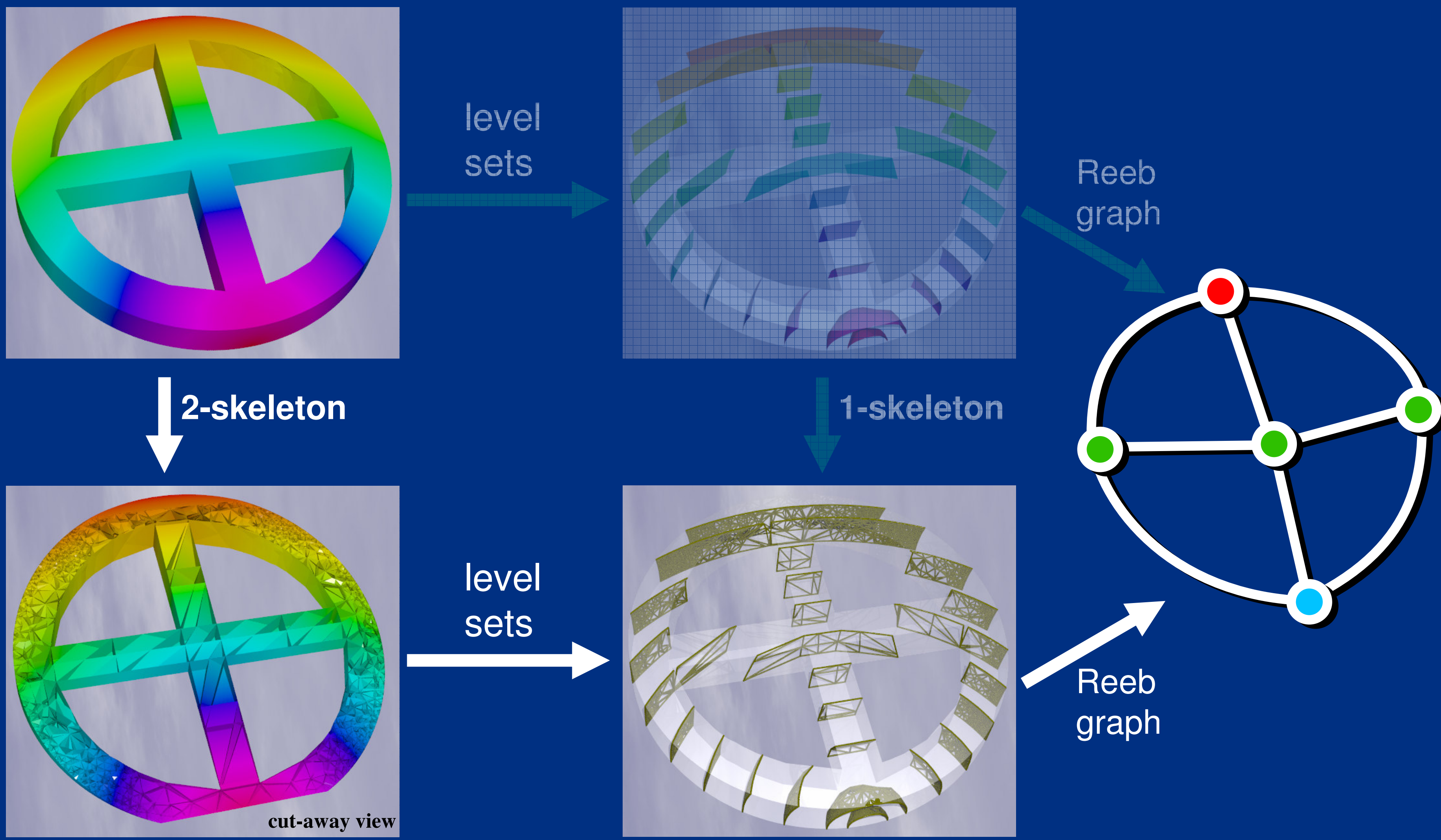

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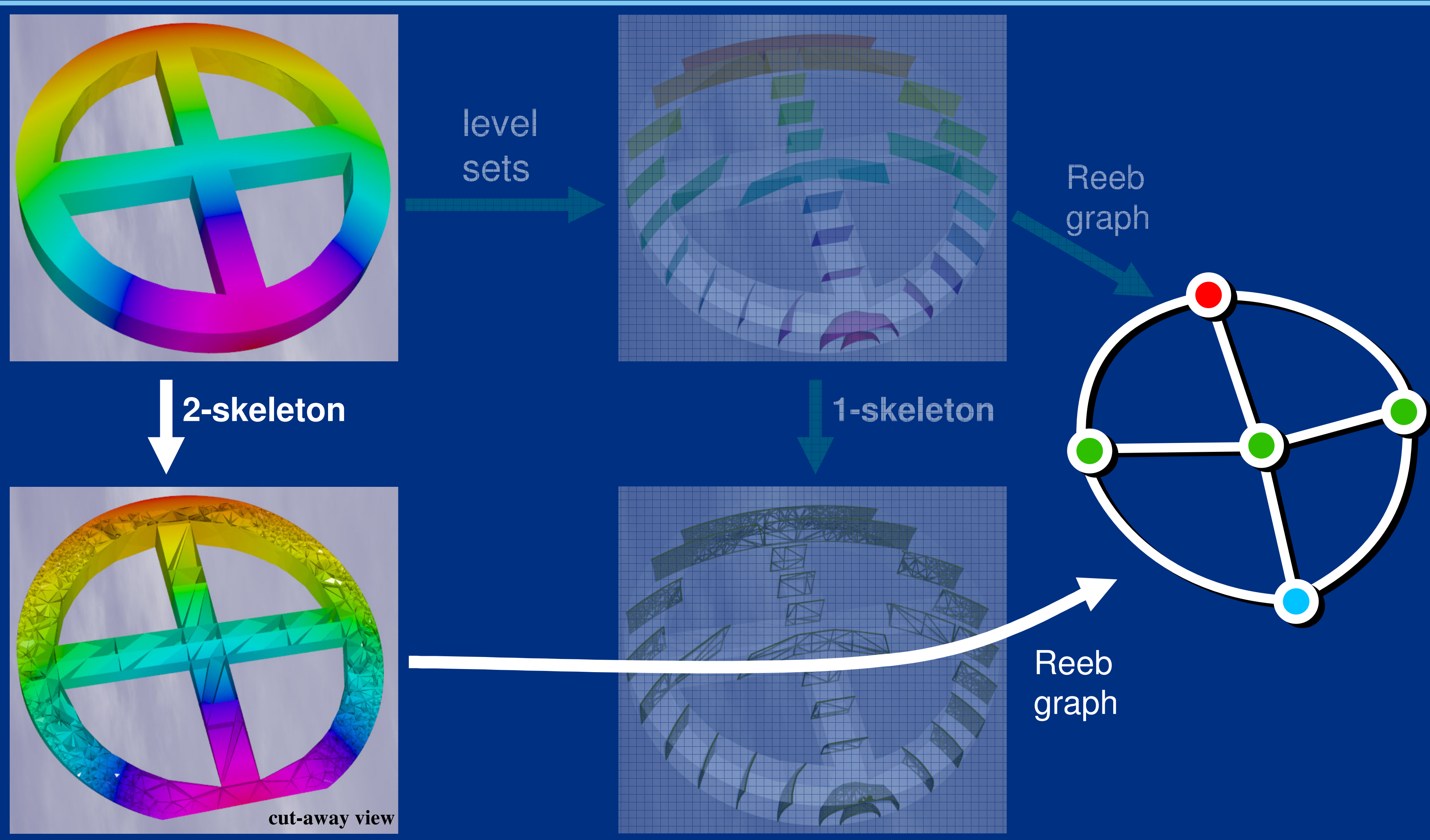

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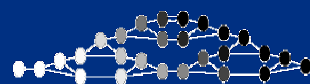
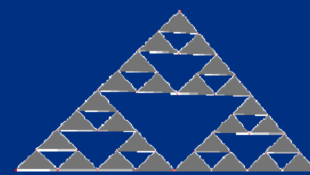
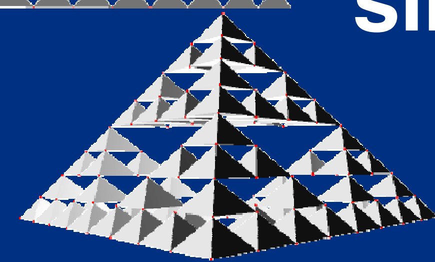
2D:

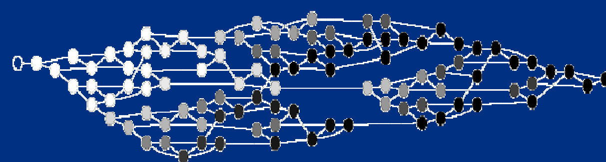






Sierpinski simplex

3D:



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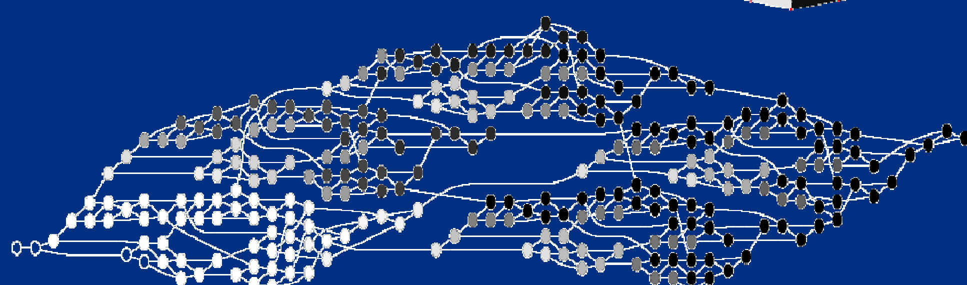

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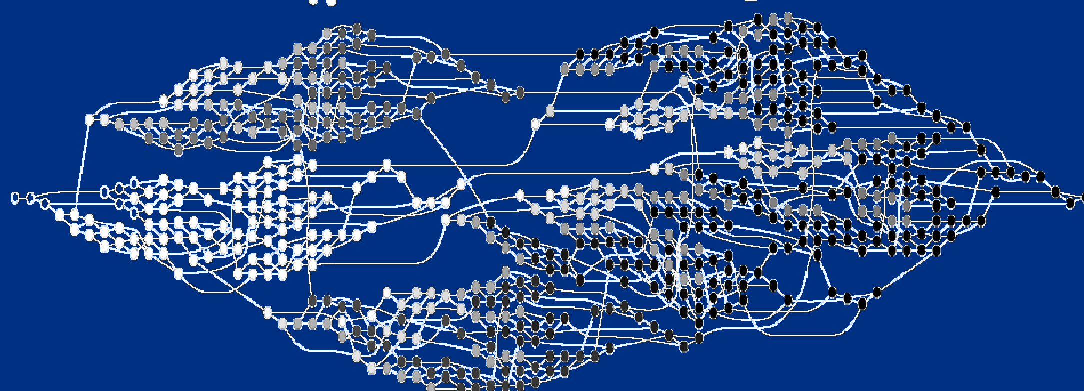

Practical Tests Confirm Robustness and Show Good Scalability